Critical Thinking in Pumping Test Interpretation

Foundations of pumping test interpretation: 2. Interpretation of pumping tests in real aquifers

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Overview

A key assumption underlying almost all solutions used to match drawdown data is that the aquifer is homogeneous. However, a visit to any outcrop of soil or rock should be enough to convince any hydrogeologist that the subsurface is complex. *Real* aquifers are heterogeneous.

The underlying heterogeneity of aquifers generally gives rise to responses at individual observation wells that are variable. Inferences of aquifer properties that are drawn from analyses of the responses at individual monitoring wells are frequently inconsistent. When different estimates of aquifer properties are obtained the only definitive finding is that the conceptual model underlying the analysis is violated. In these cases, *none* of the individual estimates of transmissivity might be reliable. In these notes an approach is suggested that may make it possible to look beyond the variations in the responses of individual wells to estimate the representative average transmissivity of heterogeneous aquifers.

Outline

1. The challenge of heterogeneity
2. An approach for interpreting pumping tests in heterogeneous aquifers
3. The interpretation of pumping tests in heterogeneous aquifers: The statistically homogeneous case
4. The interpretation of pumping tests in heterogeneous aquifers: Aquifers with distinct zones of different transmissivity
5. The significance of the composite plot
6. Summary of key points
7. References

Additional reading

- Butler (1990)

1. The challenge of heterogeneity

The interpretation of pumping tests is frequently straightforward if water level changes are monitored in only one observation well. In this case, only one estimate of the transmissivity is obtained. The interpretation of pumping tests is more challenging when multiple wells are monitored. The responses at individual observation wells will generally be variable, reflecting the underlying heterogeneity of the aquifer.

The results of a typical analysis of the responses observed at two monitoring wells during a pumping test conducted in southern Ontario are shown in Figures 1 and 2. The records for each observation well are analyzed separately.

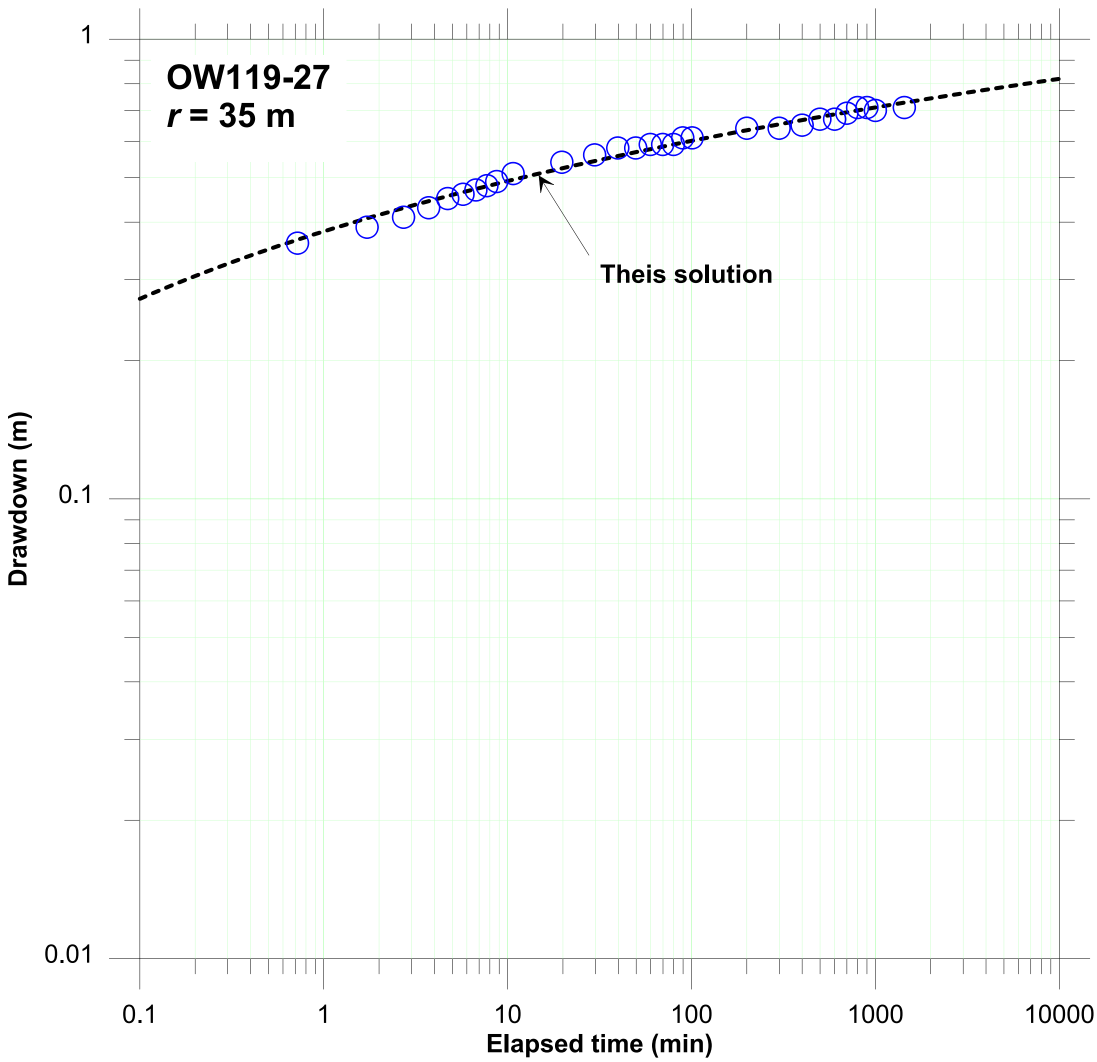

Figure 1. Match of observation well OW119-27 drawdowns with the Theis solution

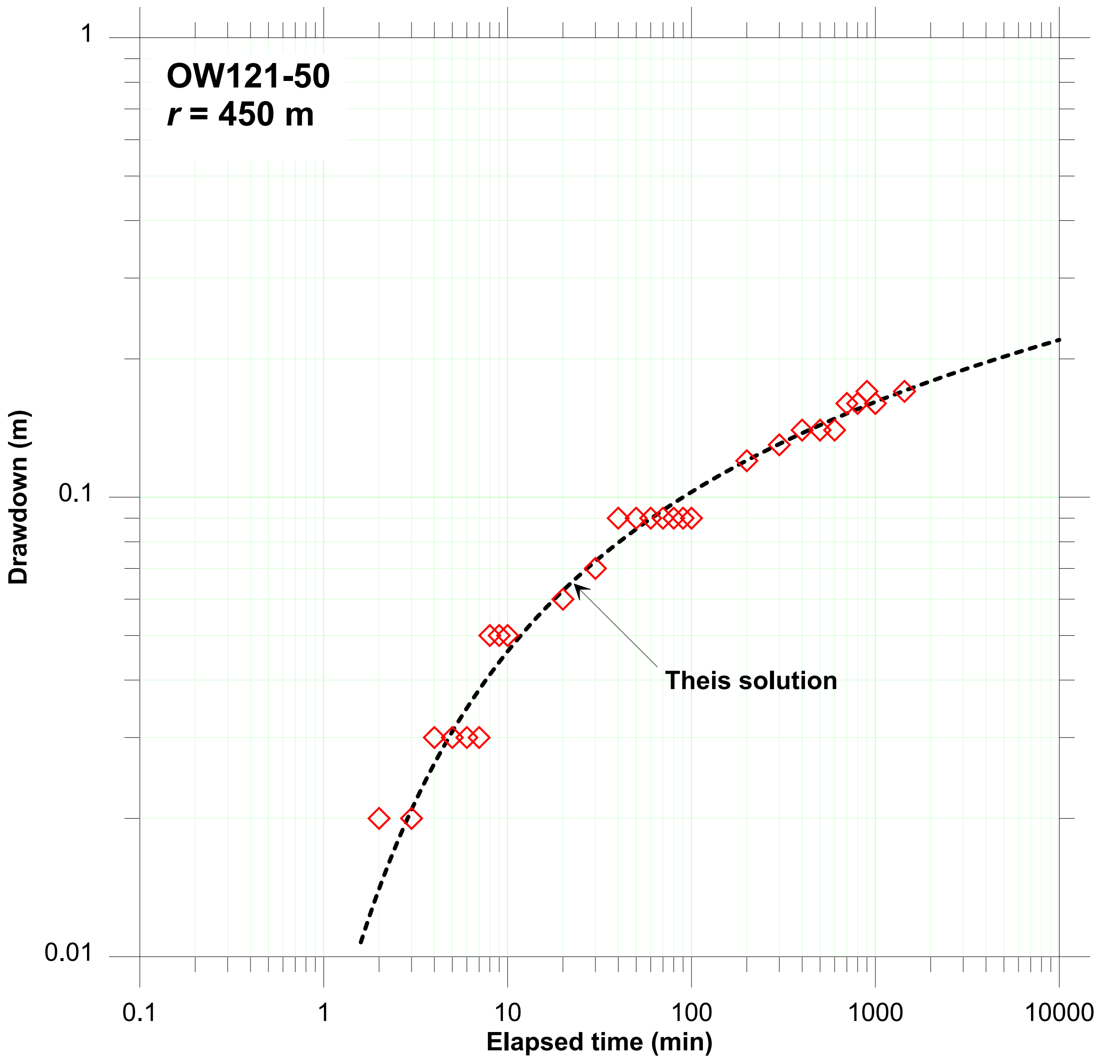

Figure 2. Match of observation well OW121-50 drawdowns with the Theis solution

It is possible to achieve relatively close matches between the observations and the Theis solution. However, as shown in Figure 3, the parameters estimated for both wells are quite different. The transmissivity estimated for OW121-50 is about double the estimate for OW119-27 and the storage coefficient is almost a factor of 100 larger.

A fundamental assumption of the Theis solution, and most other analytical models of pumping, is that the aquifer is homogeneous. The only conclusion that can be drawn from the analyses presented in Figures 1 through 3 is that the assumption of homogeneity is violated. Despite the good individual fits, the transmissivity estimates may be suspect, as the fundamental assumption of the Theis model is apparently violated. It is important to bear in mind that when inconsistent parameters are estimated among multiple wells, the estimates developed from any single observation well may not be reliable.

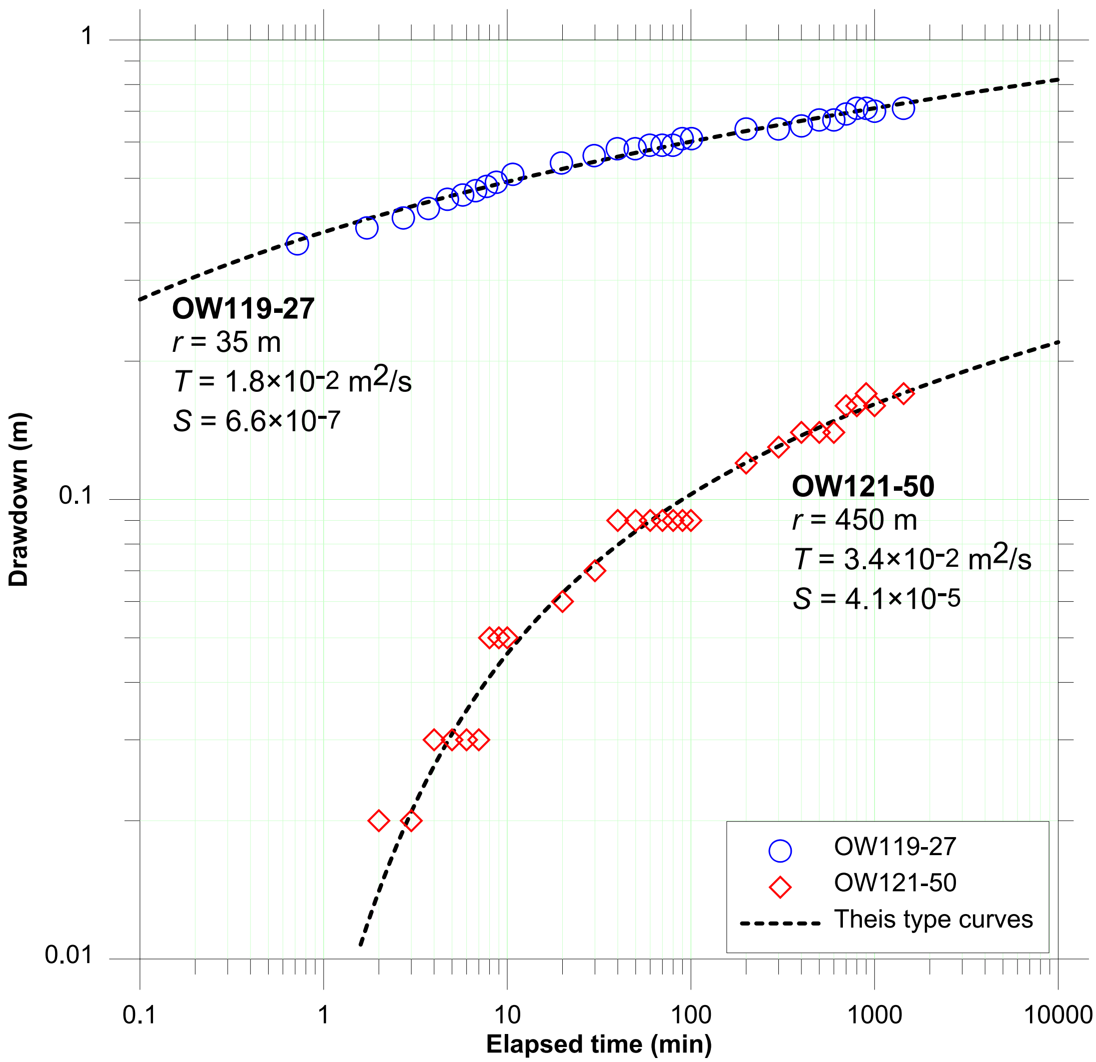

Figure 3. Summary of matches of observation well drawdowns with the Theis solution

2. An approach for interpreting pumping tests in heterogeneous aquifers

A pumping test is not conducted to characterize the details of the subsurface; rather, it is conducted to estimate the "representative bulk-average" transmissivity of a particular hydrostratigraphic unit. Here "representative" refers to an average value that provides a reliable basis for quantitative determinations at the site. This average transmissivity is also referred to as the *effective transmissivity*. Typical quantitative determinations include predictions of the amount of drawdown that will result when a production well is pumped on a sustained basis, the effects of pumping on adjacent wells or surface water bodies, and the rate at which groundwater might flow into an excavation.

In many cases we can take advantage of the strengths of the Cooper-Jacob composite analysis to identify the portion of the response that is representative of bulk-average radial flow, and to estimate a representative transmissivity from that portion of the data.

Returning to the data shown in Figures 1 and 2, the composite semilog plot for the two wells is presented in Figure 4. In an ideal aquifer, that is, in an aquifer that conforms to the assumptions of the Theis (1935) conceptual model, the drawdowns should eventually approximate a single straight line. As shown in Figure 4, the drawdowns clearly do not approximate a single straight line. These results provide the first level of diagnosis: the aquifer is <u>not</u> ideal.

An approach for interpreting the data from the two observation wells is shown in Figure 5. The dashed lines shown in the figure do not represent lines-of-best-fit. Rather, they are lines that approximate the data for both wells constructed with the same slope. Recalling the Cooper-Jacob straight-line formula for the transmissivity, parallel lines on a semilog plot yield the same estimate of transmissivity:

$$T = 2.303\,\frac{Q}{4\pi}\,\frac{1}{SLOPE}$$

The transmissivity estimated from the common slope is interpreted as the bulk-average transmissivity. The dashed lines yield different estimates of the storage coefficient; this inconsistency is interpreted as an indication of aquifer heterogeneity. In the next two sections we examine whether the proposed approach may be appropriate for the interpretation of pumping tests in heterogeneous aquifers.

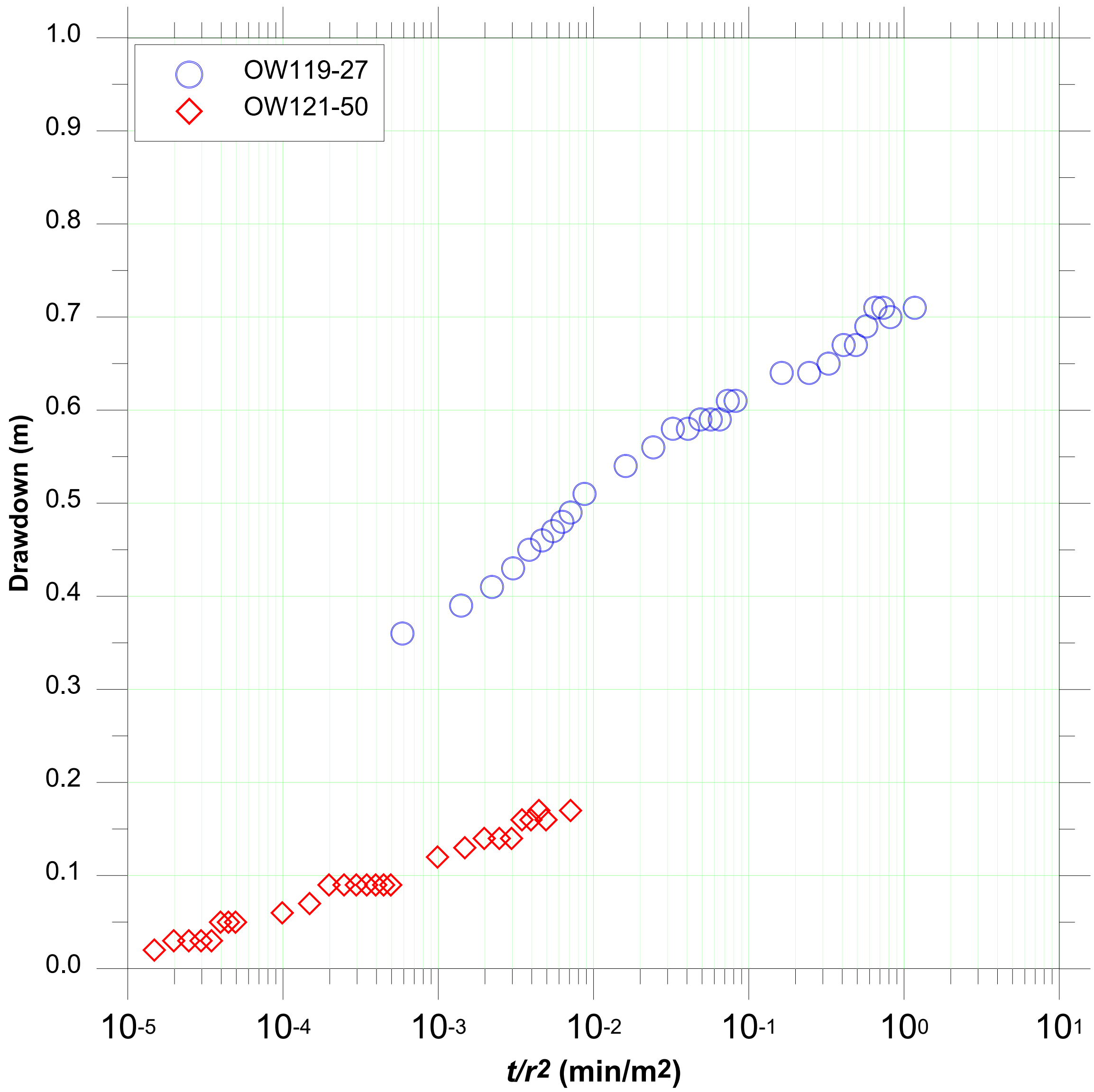

Figure 4. Composite plot of drawdowns for the two observation wells

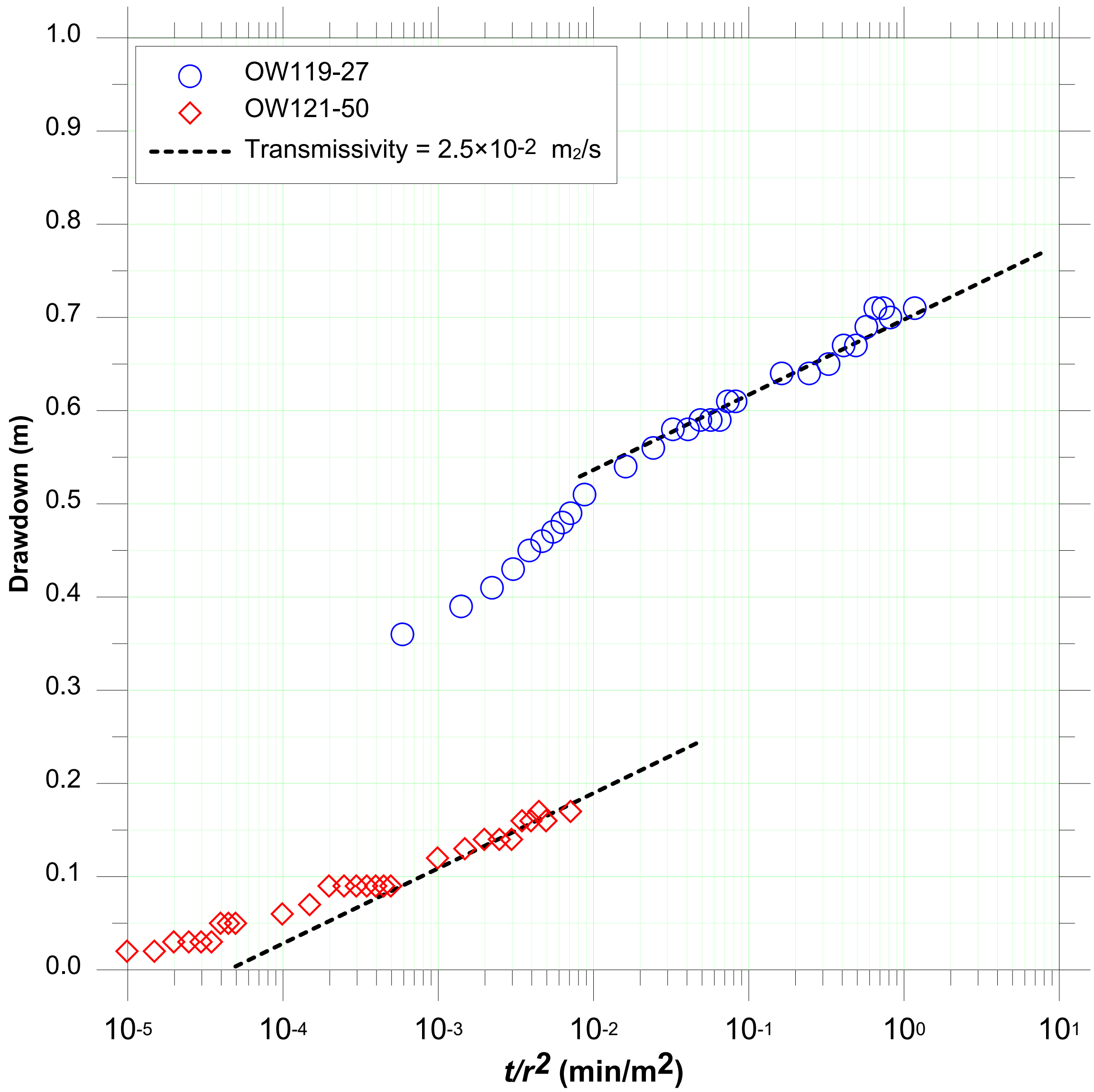

Figure 5. Cooper-Jacob composite analysis with consistent transmissivity estimates

3. The interpretation of pumping tests in heterogeneous aquifers: The statistically homogeneous case

Researchers in stochastic hydrology have investigated, through numerical simulations, the influence of aquifer heterogeneity on the responses to pumping. An important assumption in these simulations is that the aquifer is statistically homogeneous. The small-scale variations in transmissivity within a particular hydrostratigraphic unit are idealized as a random field with spatial correlation, with no large-scale trends or distinct zones with different properties.

Meier and others (1998) simulated pumping tests in heterogeneous aquifers in which the transmissivity is represented as a random correlated field with an underlying lognormal distribution. They used a plan-view numerical model to simulate pumping from a central well in the heterogeneous aquifers. The transmissivity distribution of each random field was assumed to be log-normally distributed with a geometric mean transmissivity, T_G, of 1.0 [Meier and others (1998) adopted arbitrary, but consistent units]. Variances of log-transmissivity, σ_Y^2, ranged from 0.25 to 4.0. The transmissivity distribution for a log-variance of 1.0 is reproduced in Figure 6. The detailed distribution around the pumping well is shown in the inset.

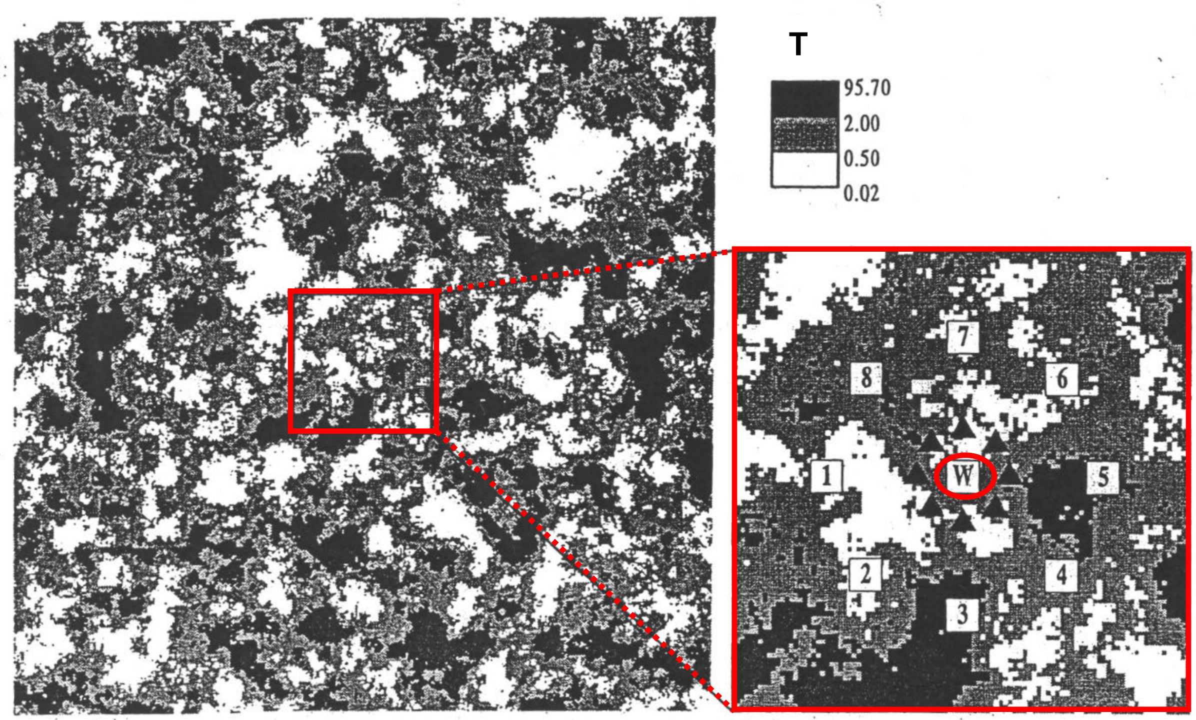

Figure 6. Random transmissivity field for σ_Y^2 = 4.0
Adapted from Meier and others (1998; Figure 8)

Case 1: $\sigma_Y^2 = 0.25$

The cumulative distribution function for $T_G = 1.0$ and $\sigma_Y^2 = 0.25$ is plotted in Figure 7. As shown in this figure, the spread of the transmissivity values about the mean value is relatively narrow.

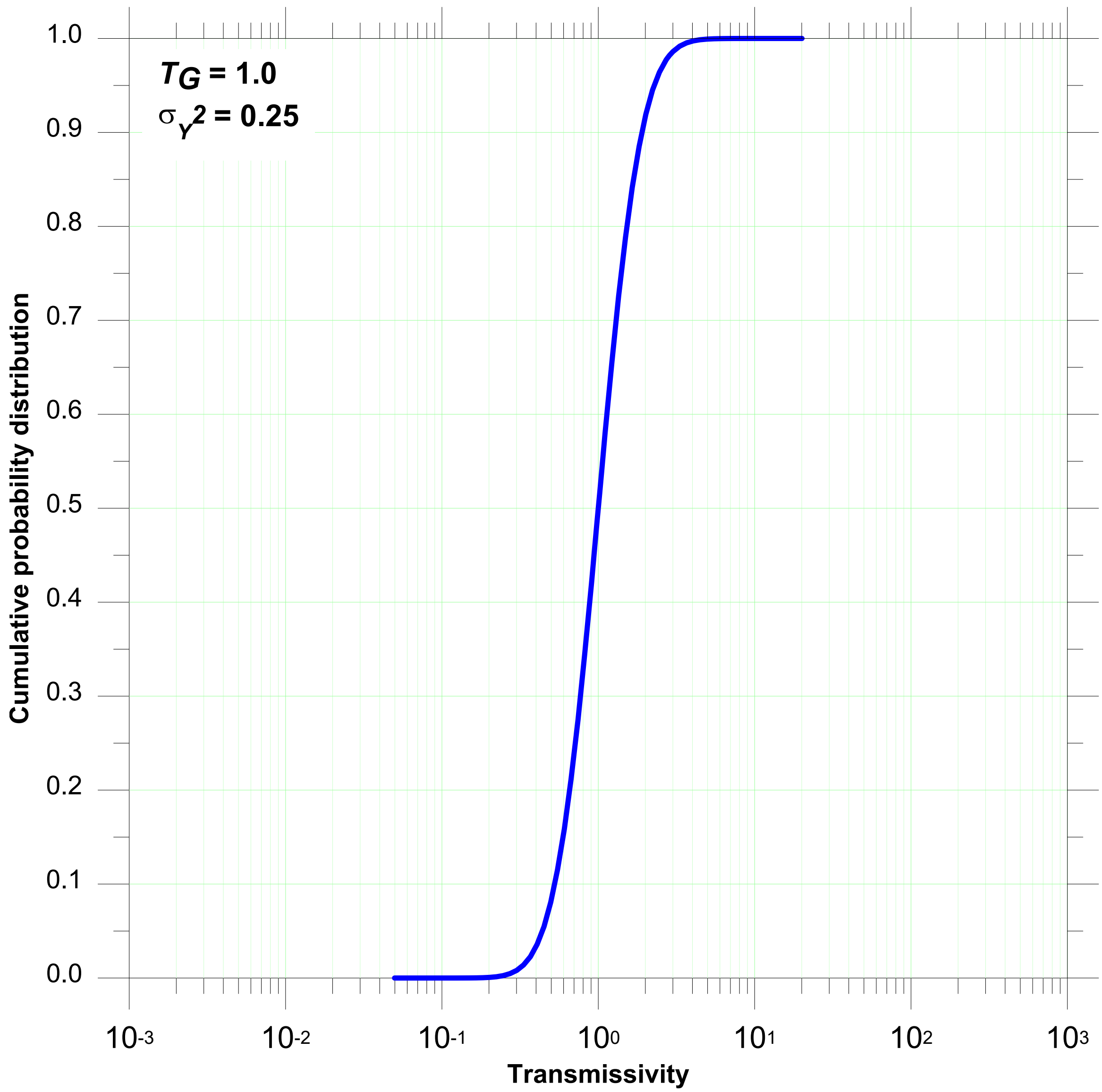

Figure 7. Cumulative distribution function of transmissivity for $\sigma_Y^2 = 0.25$

The plot of the Meier and others (1998) simulated drawdowns for $\sigma_Y^2 = 0.25$ is reproduced in Figure 8. The solid line shown in the figure denotes the response predicted for an aquifer that has a uniform transmissivity given by the geometric mean of the random field, T_G. The individual time-drawdown records at distances of 10 and 30 are approximately parallel to each other and to the lines calculated for a uniform transmissivity. This implies that consistent estimates of transmissivity will be obtained from Cooper-Jacob semilog straight-line analyses of the individual time-drawdown records.

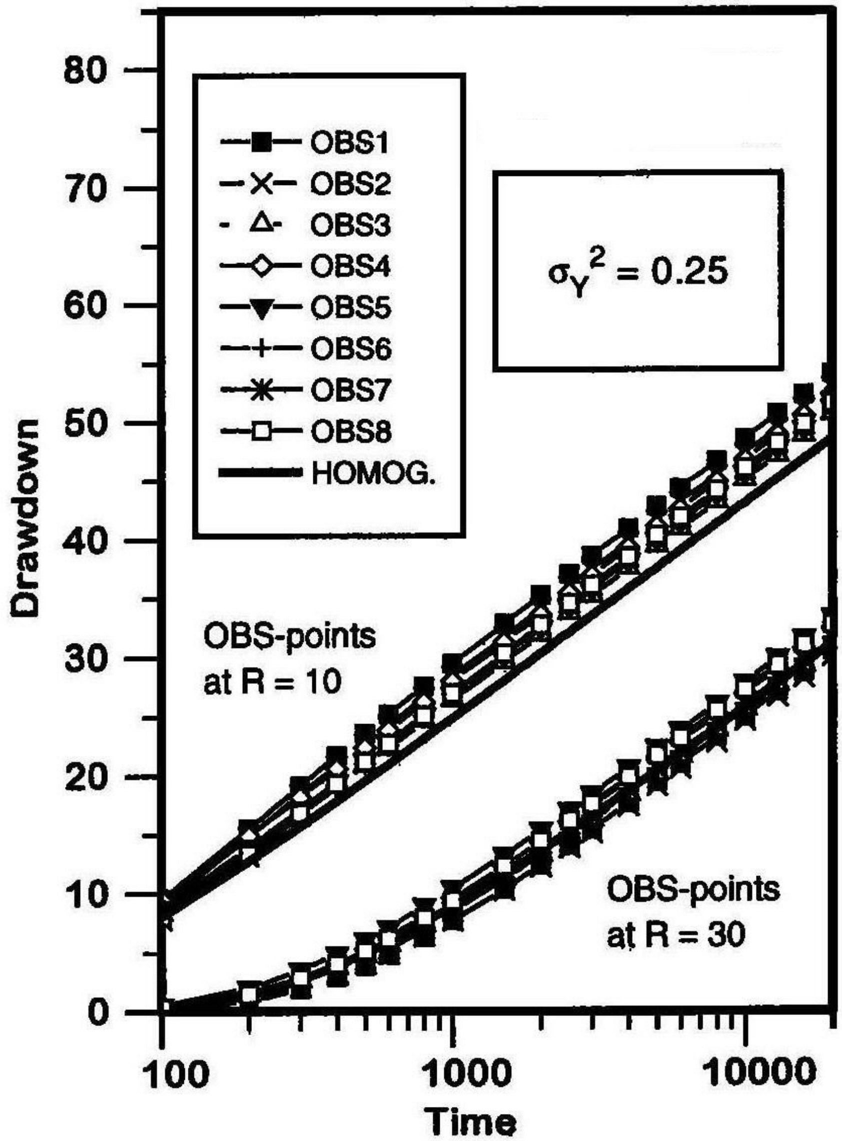

Figure 8. Simulated drawdowns for σ_Y^2=0.25
Reproduced from Meier and others (1998; Figure 8)

The simulated drawdowns for $\sigma_Y^2 = 0.25$ have been digitized and are assembled on a composite plot in Figure 9. The drawdowns from all of the wells approximate a single straight line. As shown in Figure 10, for this case of a relatively small variance of log-transmissivity, the simulated responses for the individual monitoring locations can be matched closely with the Theis solution evaluated with the geometric mean transmissivity, $T_G = 1.0$.

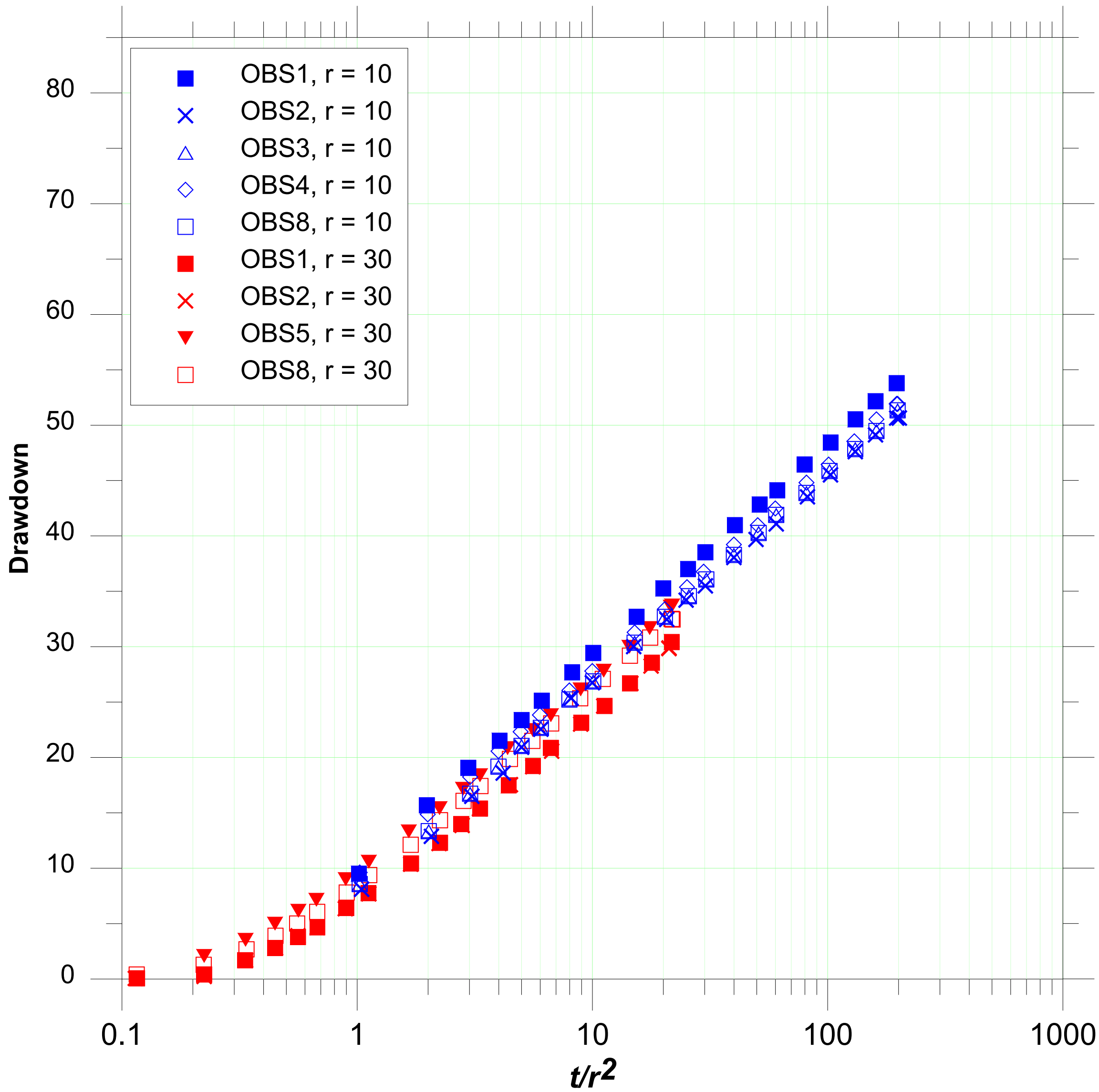

Figure 9. Composite plot of drawdowns for σ_Y^2=0.25

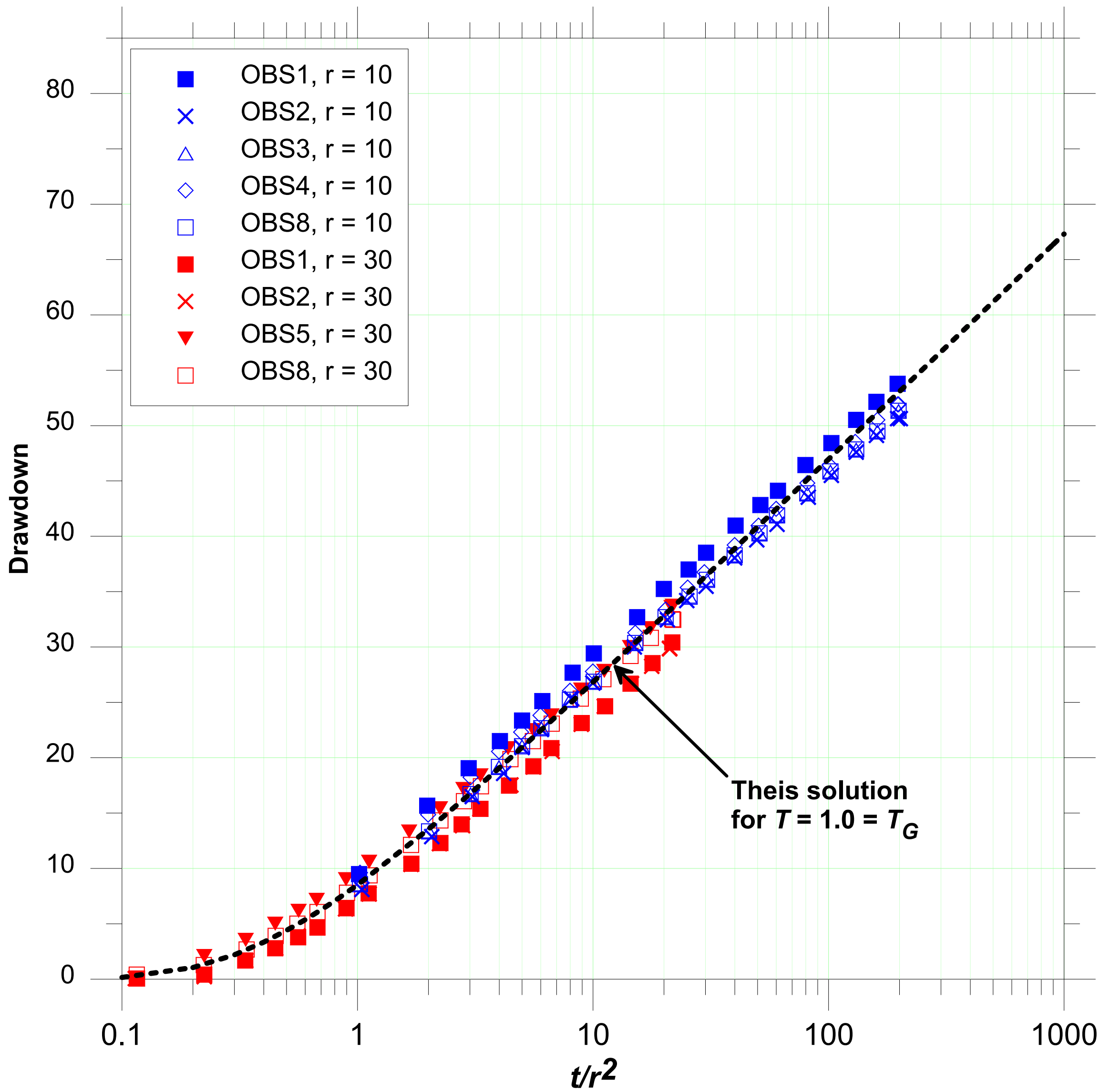

Figure 10. Composite plot of drawdowns for σ_Y^2=0.25 with Theis solution for T_G

Case 2: $\sigma_Y^2 = 4.0$

The cumulative distribution function for $\sigma_Y^2 = 4.0$ is plotted in Figure 11. The cumulative distribution function for σ_Y^2=0.25 is also shown for comparison. The distribution for a variance of 4.0 is relatively broad; the point values of transmissivity vary over about four orders of magnitude.

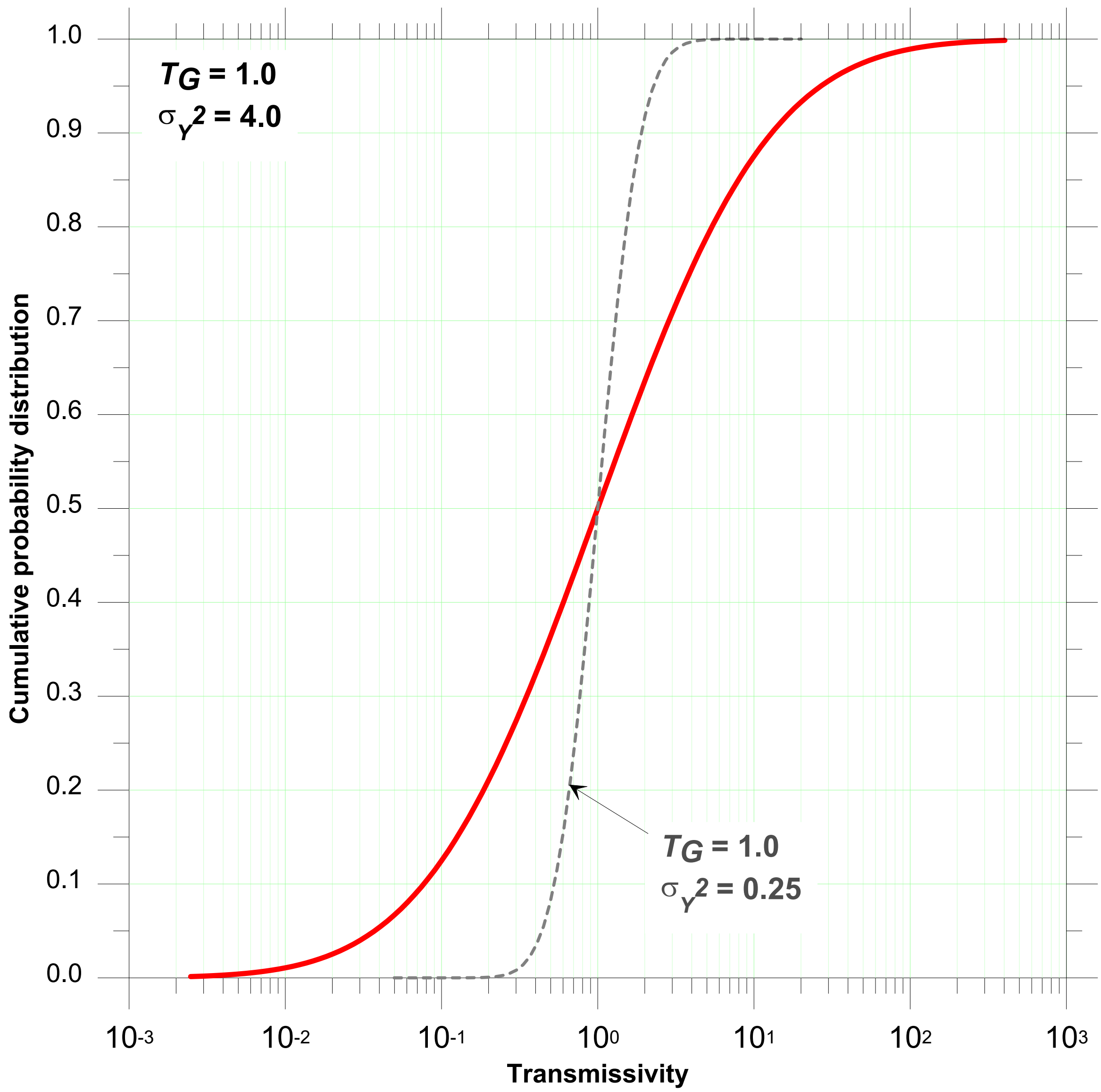

Figure 11. Cumulative distribution function of transmissivity for σ_Y^2 = 4.0

The Meier and others (1998) simulated drawdowns for $\sigma_Y^2 = 4.0$ are shown in Figure 12. There is a significant spread in the drawdowns at the different observation wells located the same distance from the pumping well. The different drawdowns are characteristic of a heterogeneous aquifer.

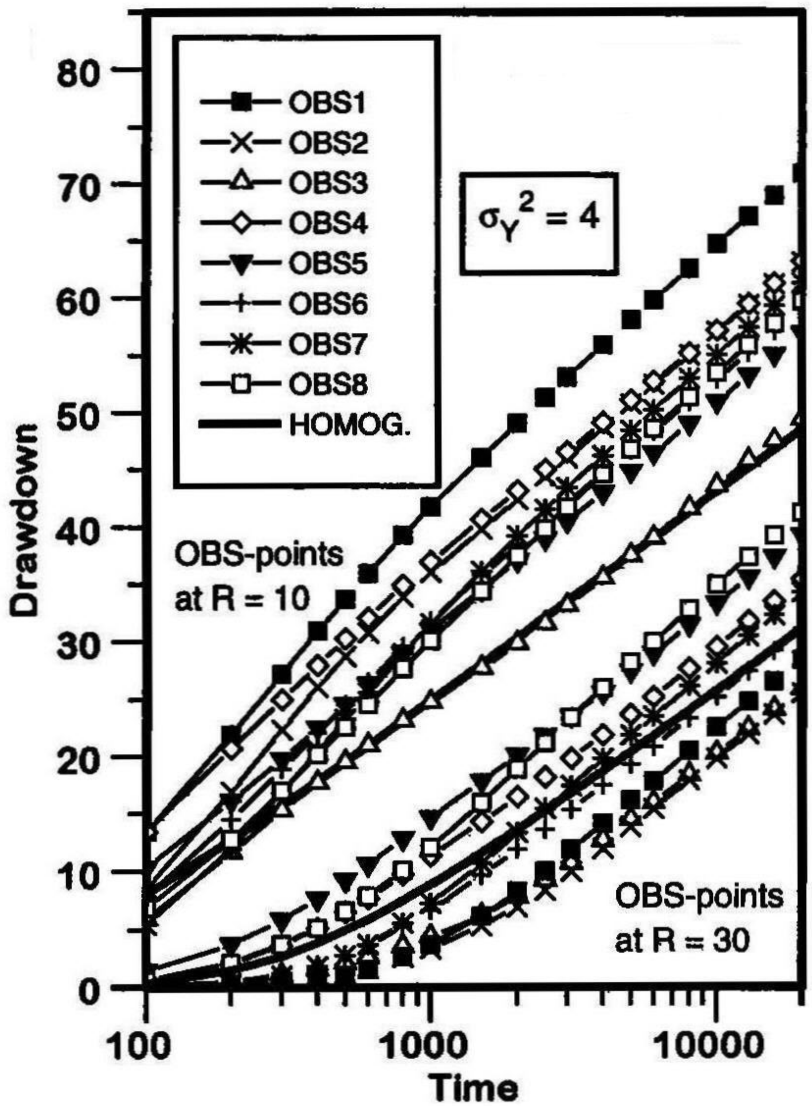

Figure 12. Simulated drawdowns for σ_Y^2=4.0
Reproduced from Meier and others (1998; Figure 8)

The simulated drawdowns for σ_Y^2= 4.0 are assembled on a composite plot in Figure 13. In contrast to the simulation results for σ_Y^2= 0.25, the individual time-drawdown records do not converge on a single line on the semilog plot. However, it is important to note that the responses of the individual wells do exhibit similar later-time slopes. Cooper-Jacob analyses based on the later-time portions of the records of the individual time-drawdown records will yield similar transmissivities but different storativities.

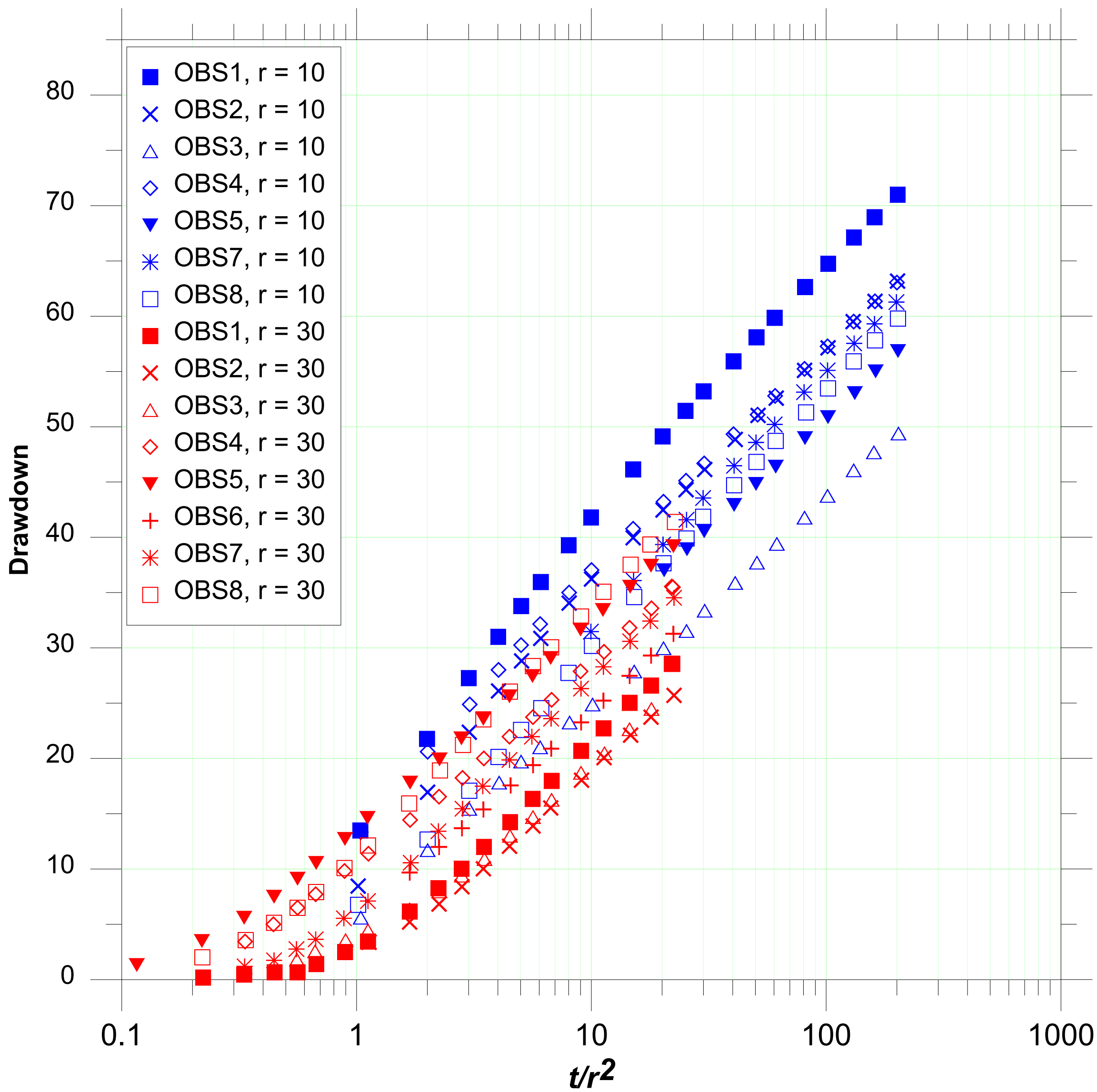

Figure 13. Composite plot of drawdowns for σ_Y^2=4.0

For the case of $\sigma_Y^2 = 4.0$, the drawdowns for the individual observation locations exhibit considerable scatter in their absolute magnitudes. However, as shown in Figure 14 the trends in the simulation results for the individual wells are roughly consistent with a bulk-average transmissivity corresponding to the geometric mean transmissivity.

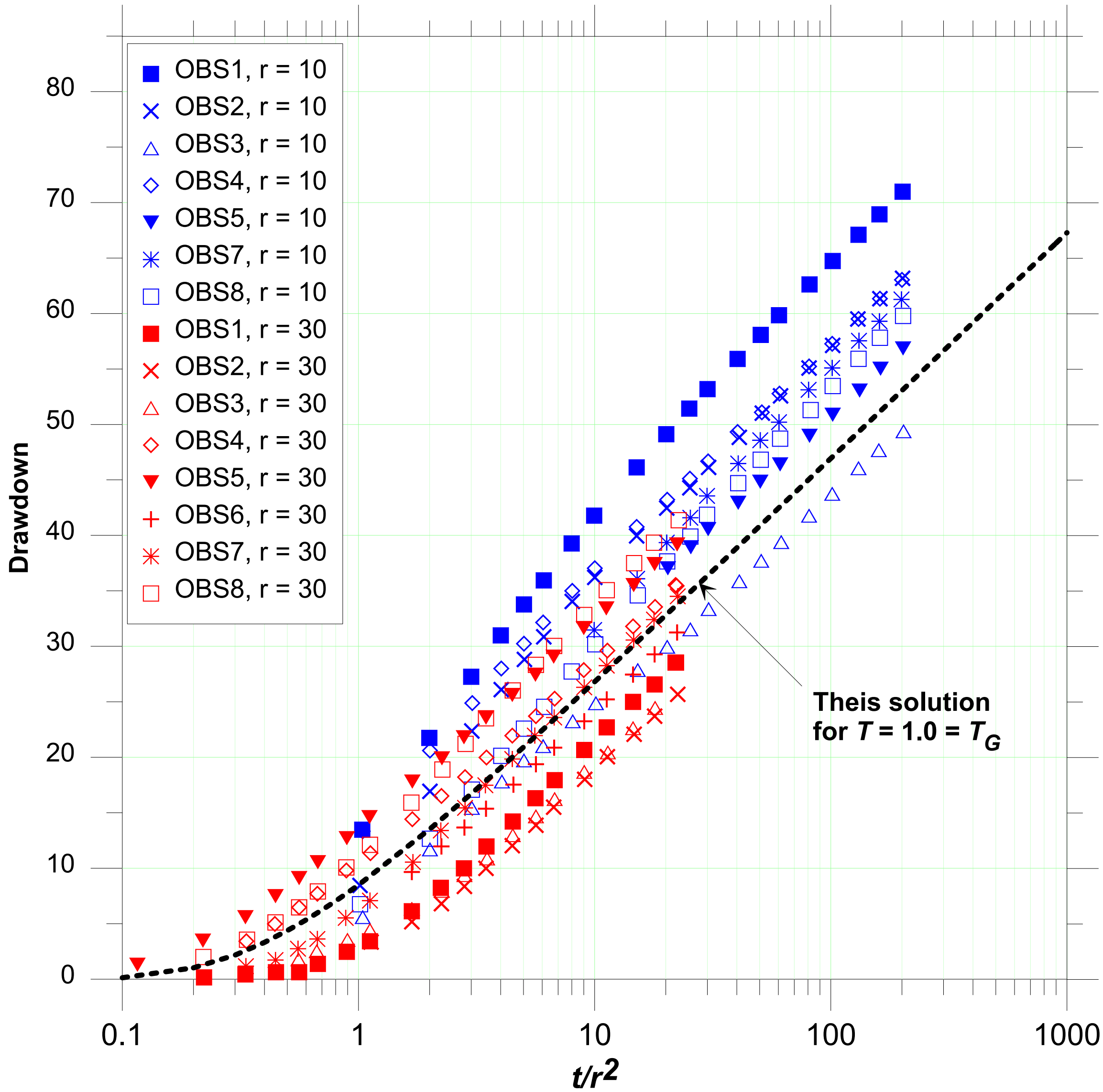

Figure 14. Composite plot of drawdowns for σ_Y^2=4.0 with Theis solution for T_G

It was indicated previously that when inconsistent parameters are estimated among multiple wells, the estimates developed from any single observation well may not be reliable. The results from two wells located the same distance from the pumping well highlight this important point. As shown in Figure 15, the simulated drawdowns for Point #8 are significantly larger than at Point #1. Point #8 is in fact located in a portion of the aquifer with relatively high transmissivity that has a direct hydraulic connection with the pumping well. The drawdowns at Point #1 are smaller; however, contrary to what would be inferred by matching the drawdowns with the Theis solution, the well is in a portion of the aquifer with relatively low transmissivity that does not have a direct connection to the pumping well.

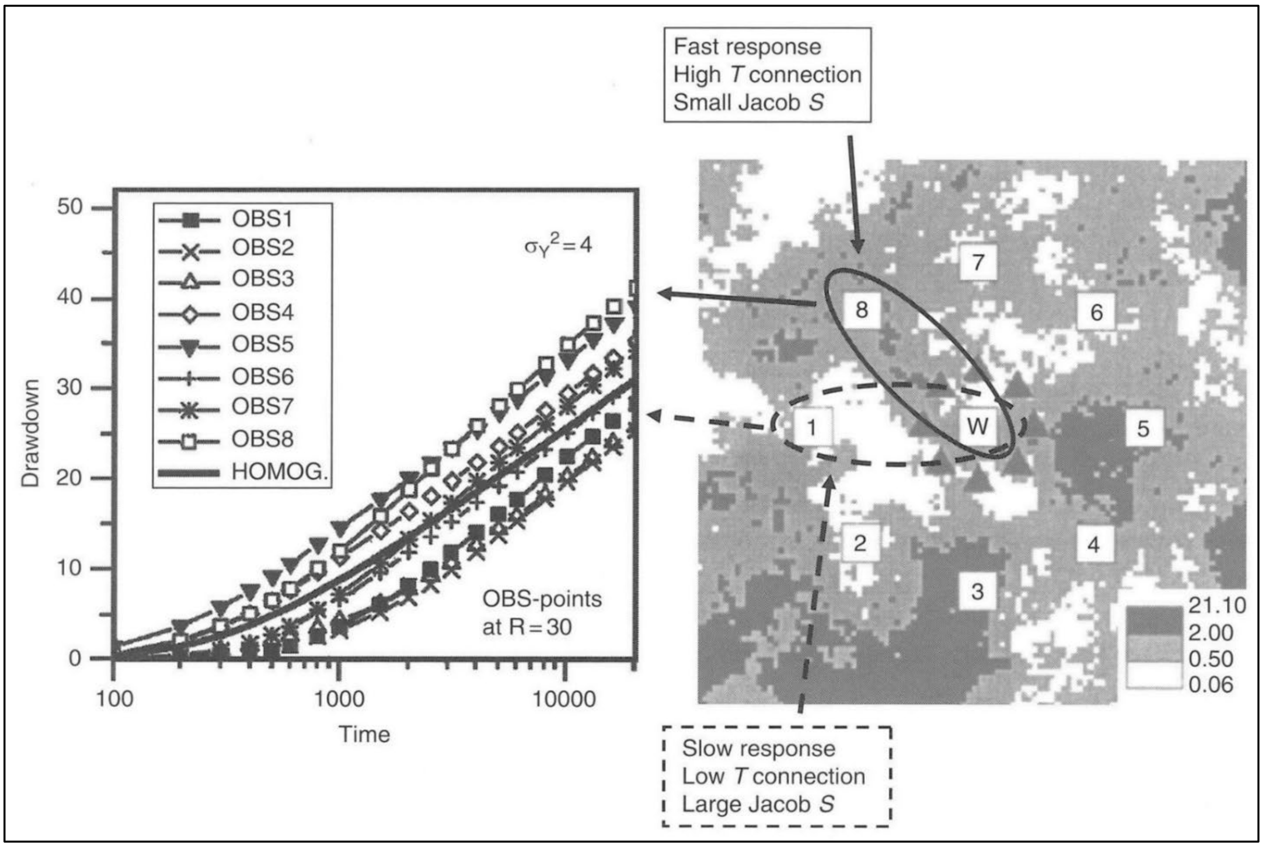

Figure 15. Examination of drawdowns simulated at two locations

Tentative conclusion from the random-field simulations:

The results of the simulations of Meier and others (1998) suggest that it is possible to estimate an effective transmissivity from a pumping test in a synthetic homogeneously heterogeneous aquifer, even for aquifers in which the degree of heterogeneity is relatively large.

Sánchez-Vila and others (1999) followed the numerical experiments of Meier and others (1998) with a theoretical analysis that examined in more detail what can be obtained from the Cooper-Jacob analysis. Their theoretical analyses confirmed that the estimated transmissivities for different observation wells tend to converge to a single value, which for a log-transformed field of transmissivity values corresponds to the geometric mean of the underlying random field.

4. The interpretation of pumping tests in heterogeneous aquifers: Aquifers with distinct zones of different transmissivity

Dr. James J. Butler and his colleagues at the Kansas Geological Survey have developed analytical solutions for an important class of problems involving transient flow to a well in a heterogeneous aquifer that has distinct zones (Butler, 1988; Butler and Liu, 1991; Butler and Liu, 1993). Butler and Liu (1993) derived a solution for pumping in an aquifer that contains a circular zone with properties that are different from the rest of the formation. The conceptual model is illustrated in Figure 16. The circular zone is referred to here as a *pod*.

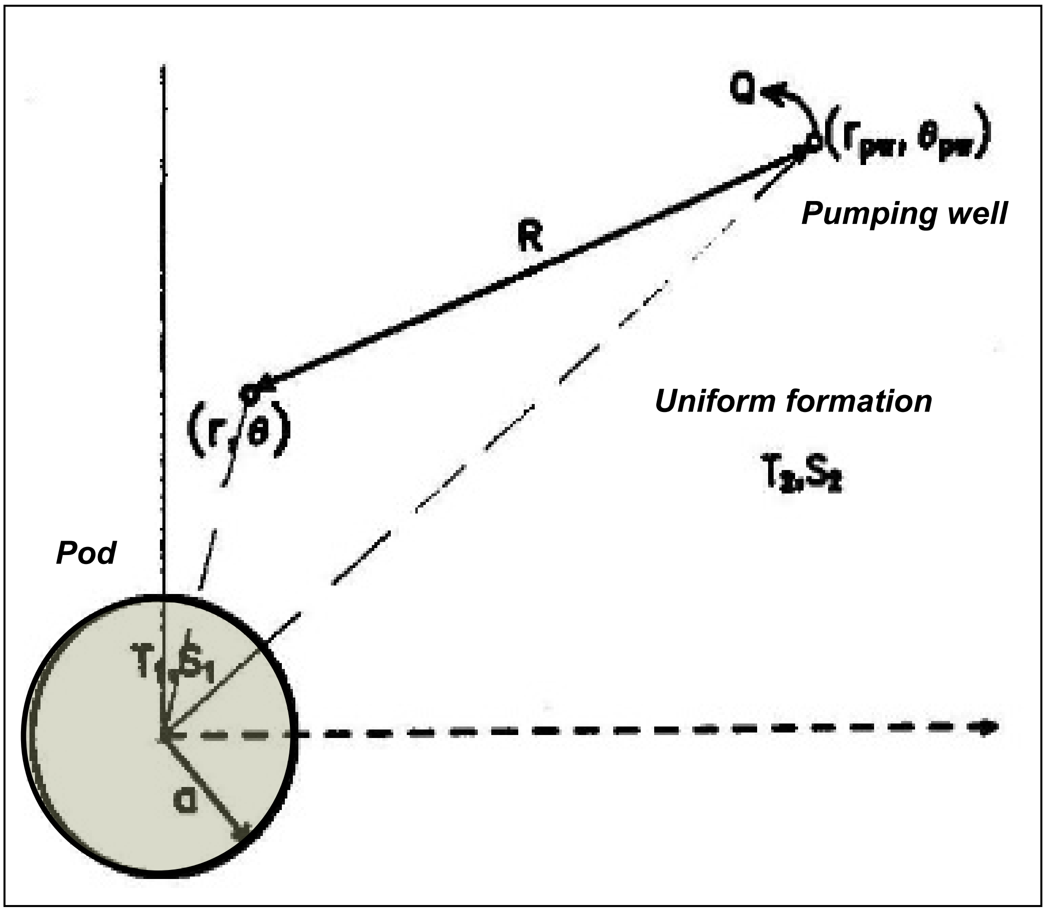

Figure 16. Conceptual model for the Butler and Liu (1993) pod solution

The parameters for the Butler and Liu (1993) model are defined below.

Parameter	Description	Units
T_1	Transmissivity of the pod	L^2T^{-1}
S_1	Confined storage coefficient of the pod	-
T_2	Transmissivity of the formation	L^2T^{-1}
S_2	Confined storage coefficient of the formation	-
a	Radius of the pod	L
Q	Pumping rate	L^3T^{-1}

The locations of any points are defined in terms of the distance between the center of the pod and the point, r, and the angle of the ray that connects the center of the pod and the point, θ (with respect to the horizontal).

- Pumping well: r_{PW}, θ_{PW}
- Any observation well: r, θ

The solution of Butler and Liu (1993) is used here to simulate two cases. In the first case the pumping well is located within a pod. In the second case, an observation well is located within a pod.

Case 1: Pumping well located in a pod

The conceptual model for Case #1 is shown in Figure 17. The transmissivity of the formation, T_2, is 100 m^2/day. The pumping well is located at the center of a circular pod of 5 m radius that has a significantly lower transmissivity, T_1 = 0.1 m^2/day. A uniform storage coefficient $S_1 = S_2 = 5\times10^{-4}$ is assumed. Observation wells #3 and #4 are located 10 m from the pumping well. Wells #1 and #6 are located 50 m from the pumping well. As shown in Figure 17, the observation wells are located symmetrically with respect to the pd and the pumping well. All four of the observation wells are located outside of the pod. The well is pumped at a constant rate of 100 m^3/day.

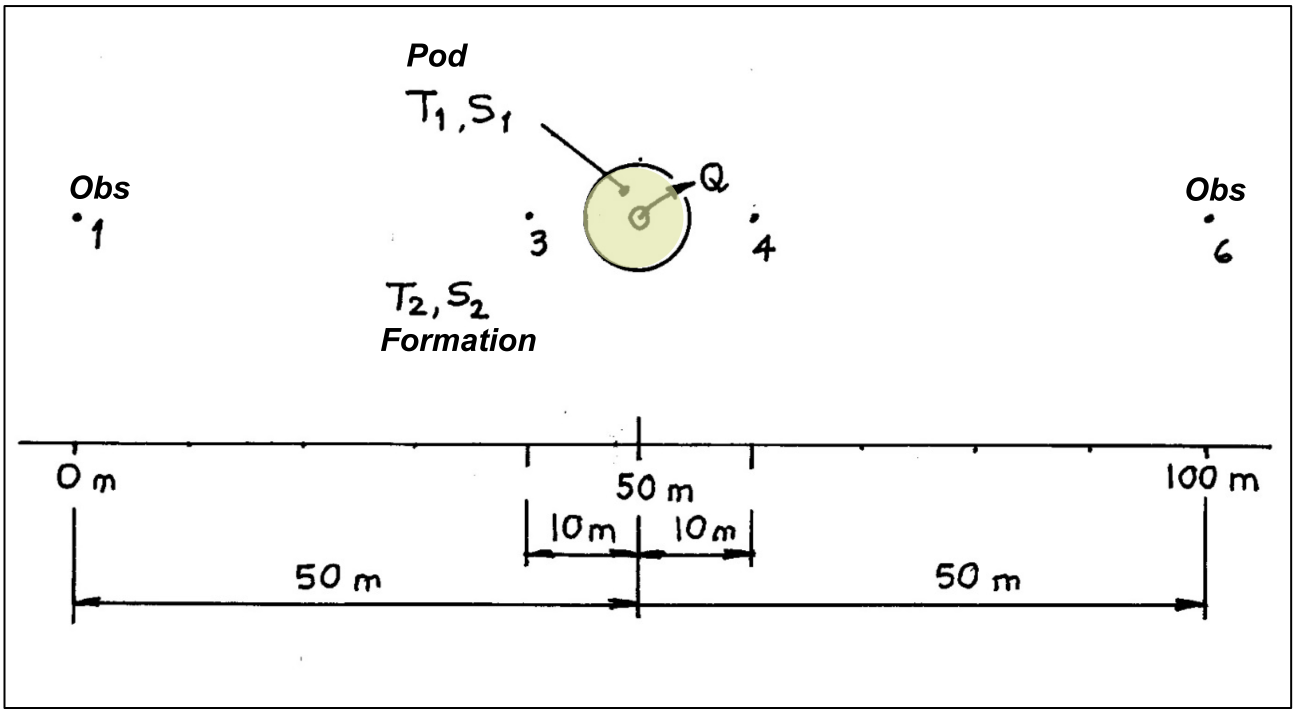

Figure 17. Conceptual model for Case #1 pod simulation

The simulated drawdowns are plotted in Figure 18. Since the observation wells are symmetric with respect to the pumping well and the pod, the drawdowns for the two observation wells 3 and 4 at distances of 10 m from the pumping well are the same, as are the drawdowns for the two observation wells 1 and 6 at distances of 50 m.

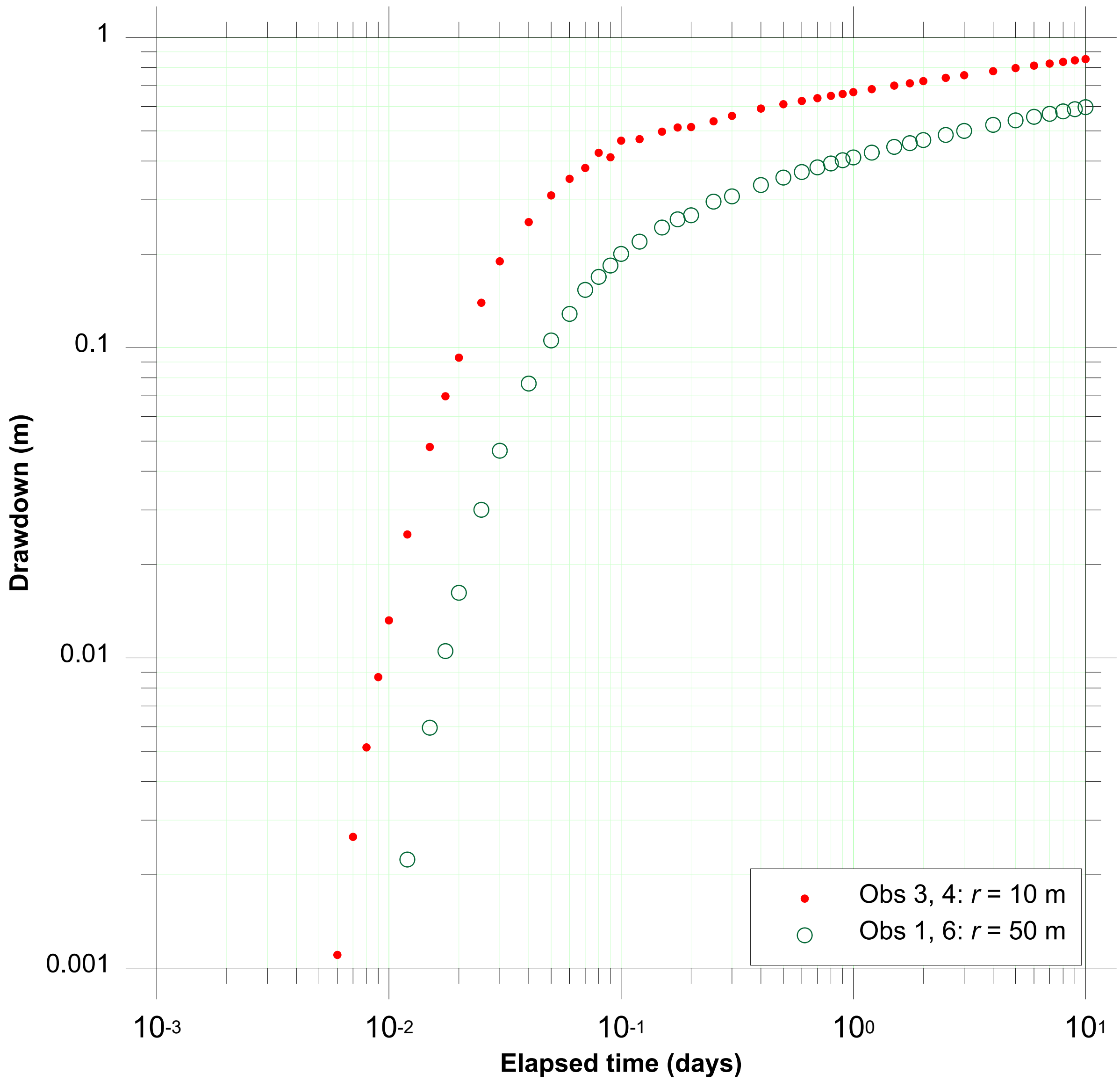

Figure 18. Simulated drawdowns at observation wells at 10 m and 50 m

The match of the Theis solution to the simulated drawdowns at the two observation wells located 10 m from the pumping well is shown in Figure 19. The dashed line represents the “best fit” obtained with a nonlinear regression routine. The match shown is a “best fit” only in a statistical sense, as the solution does not match any portion of the response particularly well. The estimated transmissivity is 64 m^2/d, which is not representative of the transmissivity of either the formation (100 m^2/d) or the pod (0.1 m^2/d).

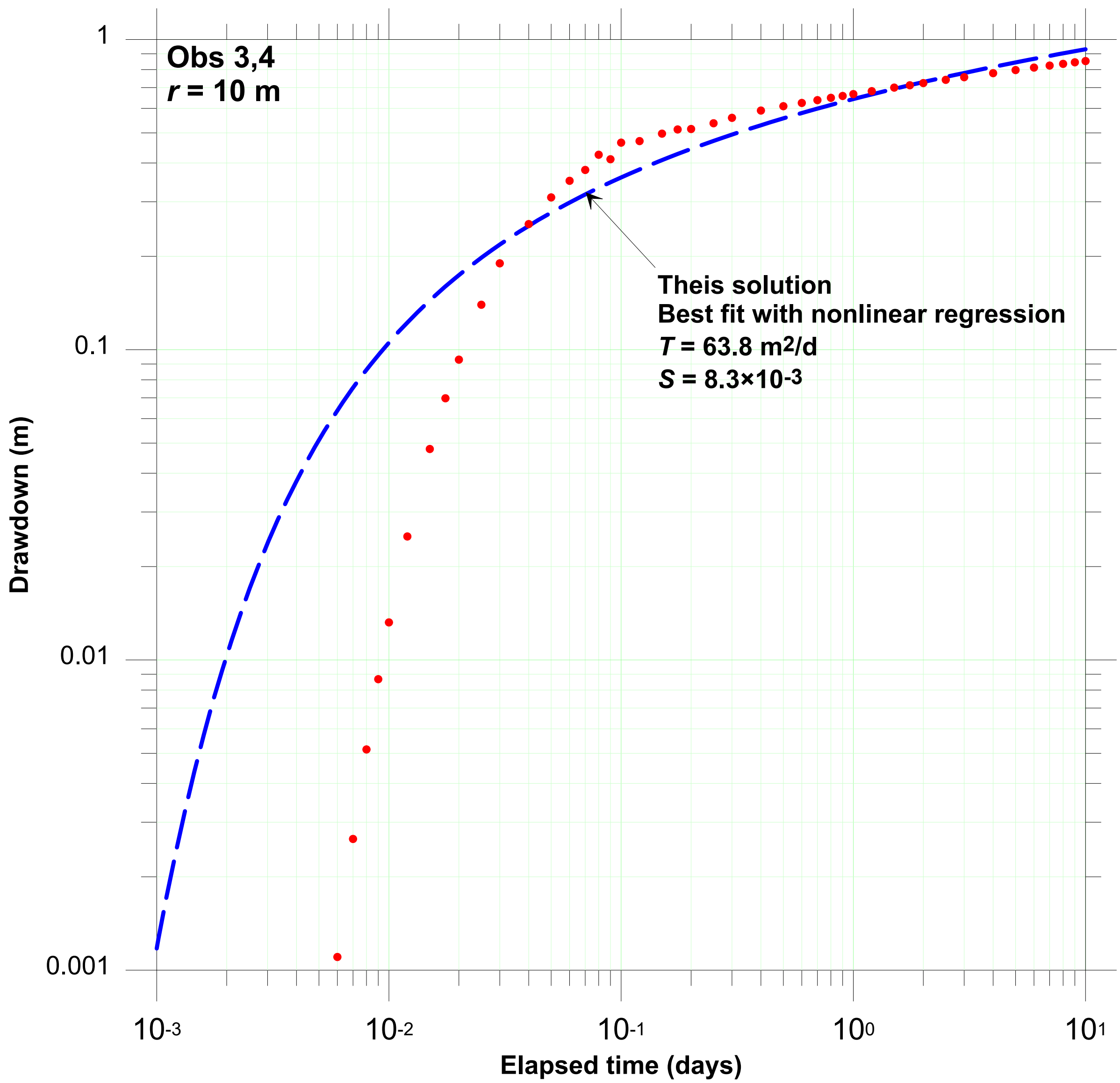

Figure 19. Match of the Theis solution to the drawdowns at observation wells at *r* = 10 m

The drawdowns calculated with the Theis solution at *r* = 10 m and the “true” formation parameters are superimposed on the simulated drawdowns in Figure 20. The Theis solution matches closely the drawdowns beyond 0.1 days; however, it is unlikely that an analyst would be willing to accept the apparently poor match to the earlier drawdowns.

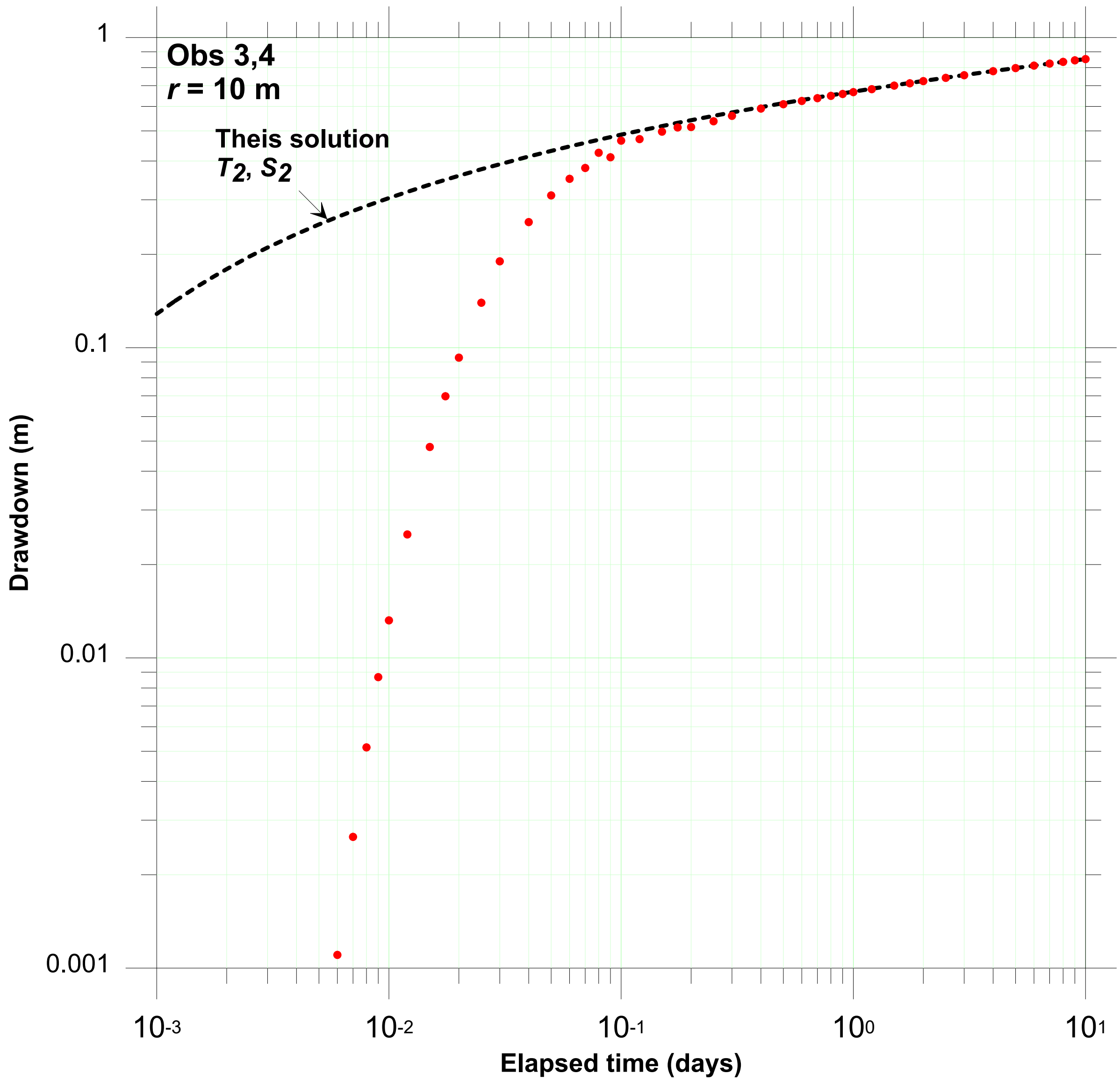

Figure 20. Theis solution with formation parameters, observation wells at *r* = 10 m

The match of the Theis solution to the simulated drawdowns at the two observation wells located 50 m from the pumping well is shown in Figure 21. The dashed line represents the "best fit" obtained with a nonlinear regression routine. The match to the early drawdowns is relatively poor, but a relatively good match to the simulated drawdowns in achieved after about 0.1 days. The fitted transmissivity is about 80% of the value specified for the formation. Although the match appears to be improved with respect to the observation wells at 10 m, the inability to match the complete drawdown record points to complexity in the aquifer.

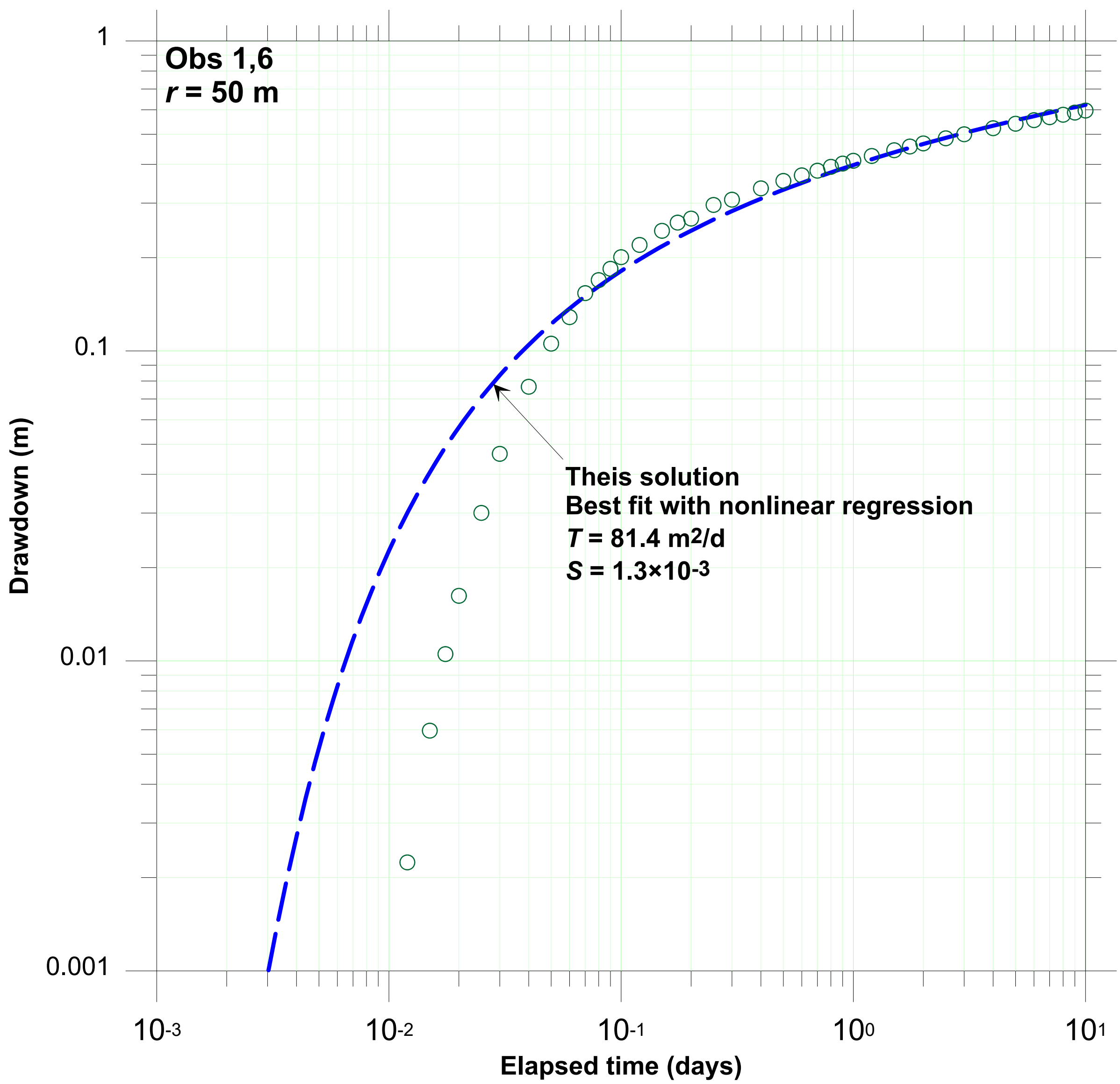

Figure 21. Match of the Theis solution to the drawdowns at observation wells at *r* = 50 m

The drawdowns calculated with the Theis solution at $r = 50$ m and the "true" formation parameters are superimposed on the simulated drawdowns in Figure 21. The results with the Theis solution match closely the last portion of the simulated drawdowns. Again it is unlikely that an analyst would be willing to accept the poor match to much of the drawdowns. If data similar to the simulated drawdowns shown in Figure 22 were obtained from an actual pumping test, and the properties of the formation were not known in advance, there is no guarantee that an analyst would recognize the correct portion of the response to match with the Theis solution.

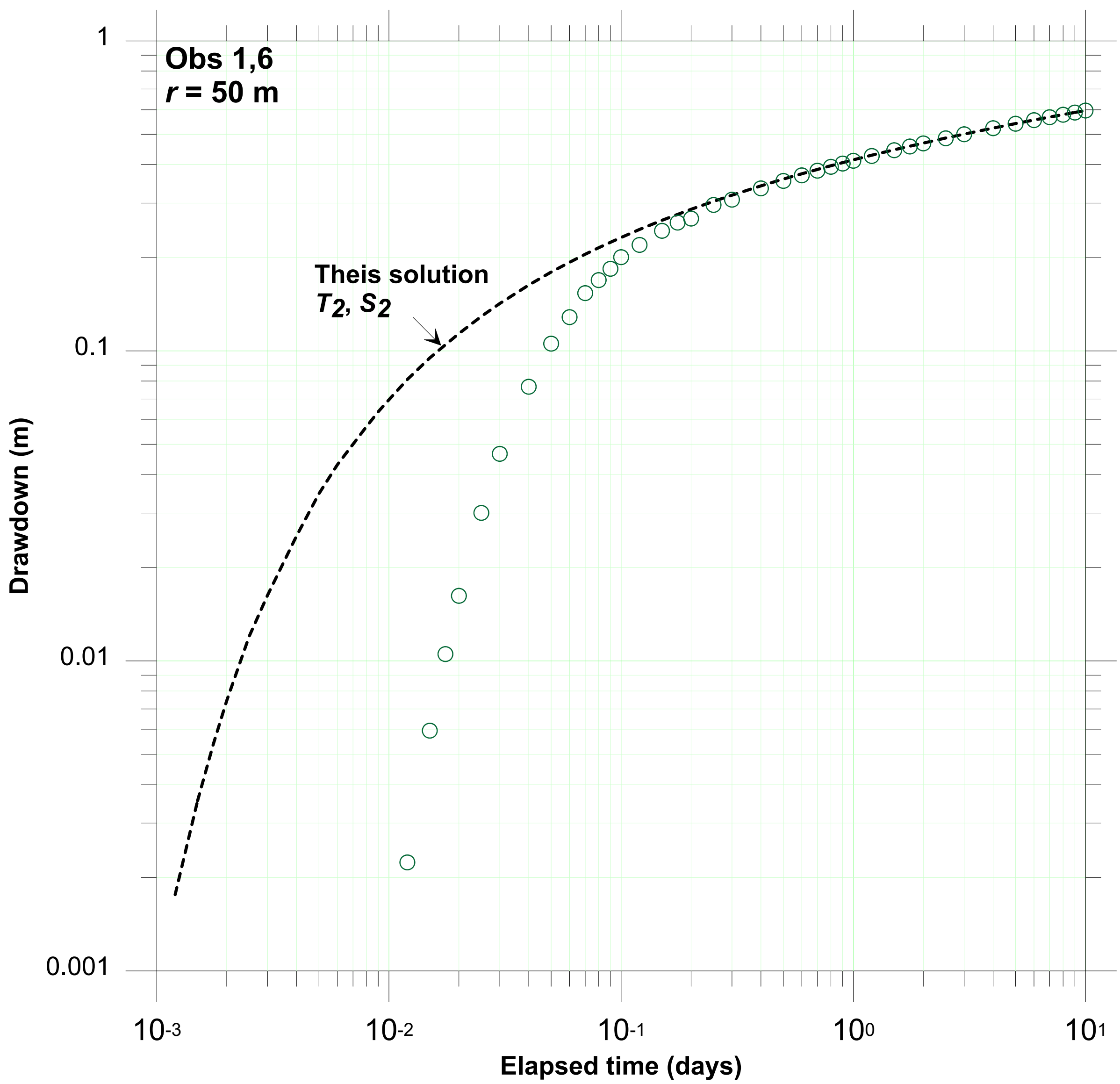

Figure 22. Theis solution with formation parameters, observation wells at *r* = 50 m

The simulated drawdowns for the observation wells are assembled in a single composite plot in Figure 23. Here the composite semilog plot shows its strengths. The convergence of the simulated drawdowns on a common later-time straight line is evident and there is no ambiguity in identifying the portion of the plot to match to obtain a consistent, representative estimate of the transmissivity of the formation.

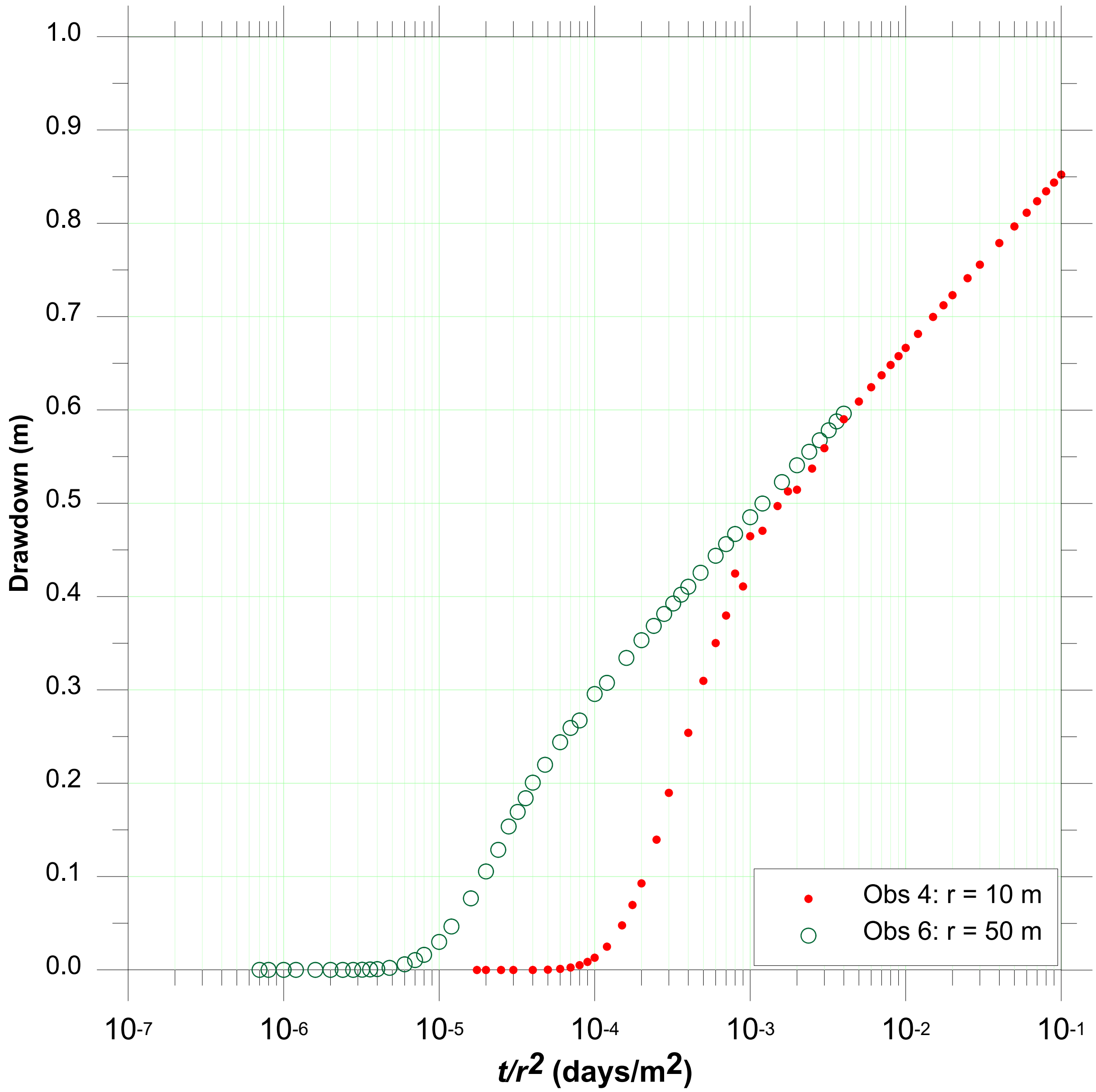

Figure 23. Semilog composite plot for Case #1 pod simulation

As shown in Figure 24, a Cooper-Jacob analysis over the common straight line portion of the two simulated responses yields an estimate of the transmissivity identical to that specified for the formation.

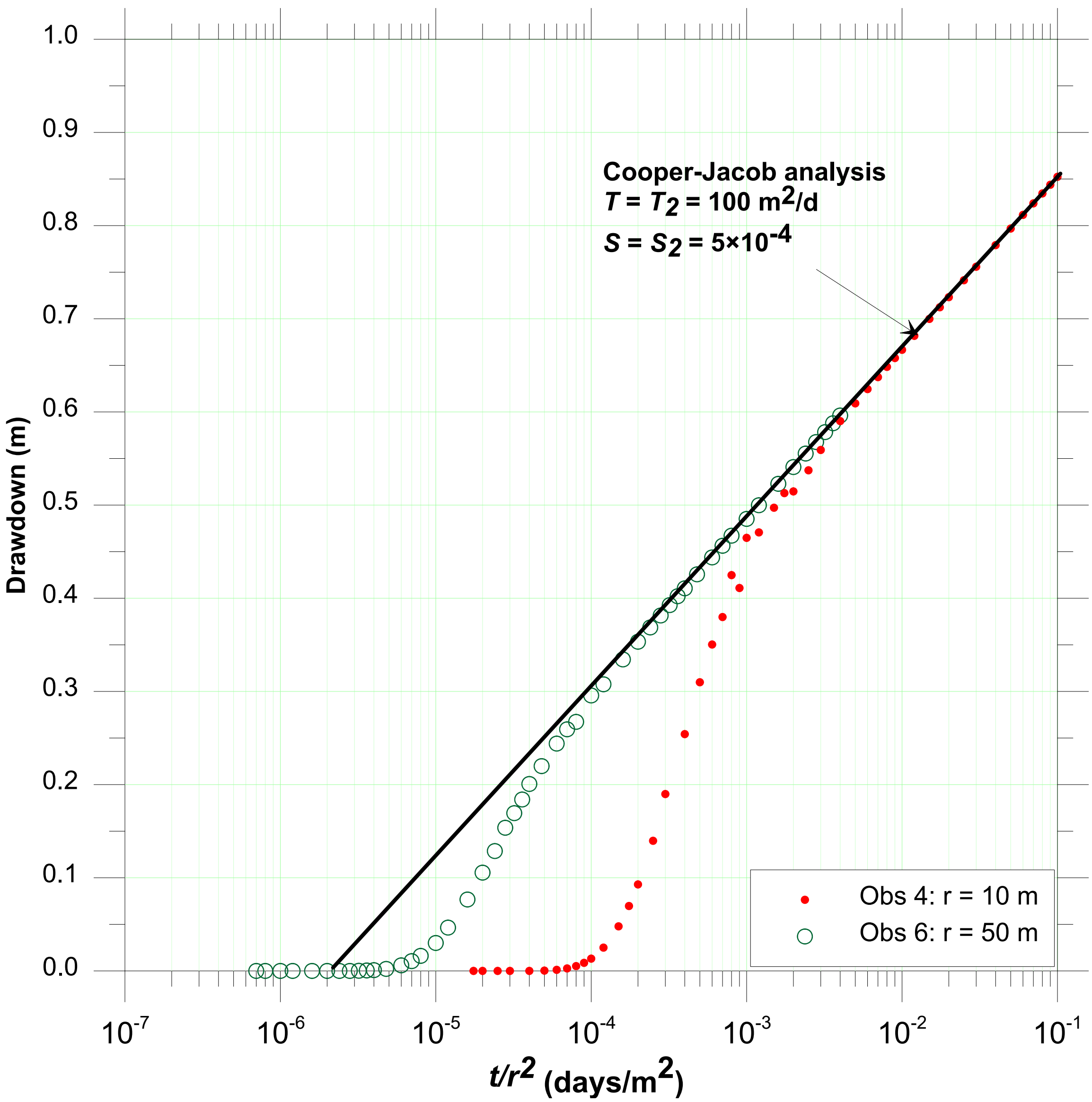

Figure 24. Semilog composite plot for Case #1 with Cooper-Jacob analysis

Case #2: Observation well located in a pod

The conceptual model for the second case is shown schematically in Figure 25. The transmissivity of the formation is 100 m^2/day (T_2). Observation wells #3 and #4 are located at the same distance from the pumping well (10 m), as are observation wells #1 and #6 (50 m). Observation well #1 is located 50 m from the pumping well, at the center of a circular pod of 10 m radius that has a transmissivity T_1 = 0.1 m^2/day. The storage coefficient is uniform, $S_1 = S_2 = 5\times10^{-4}$. The well is pumped at a constant rate of 100 m^3/day.

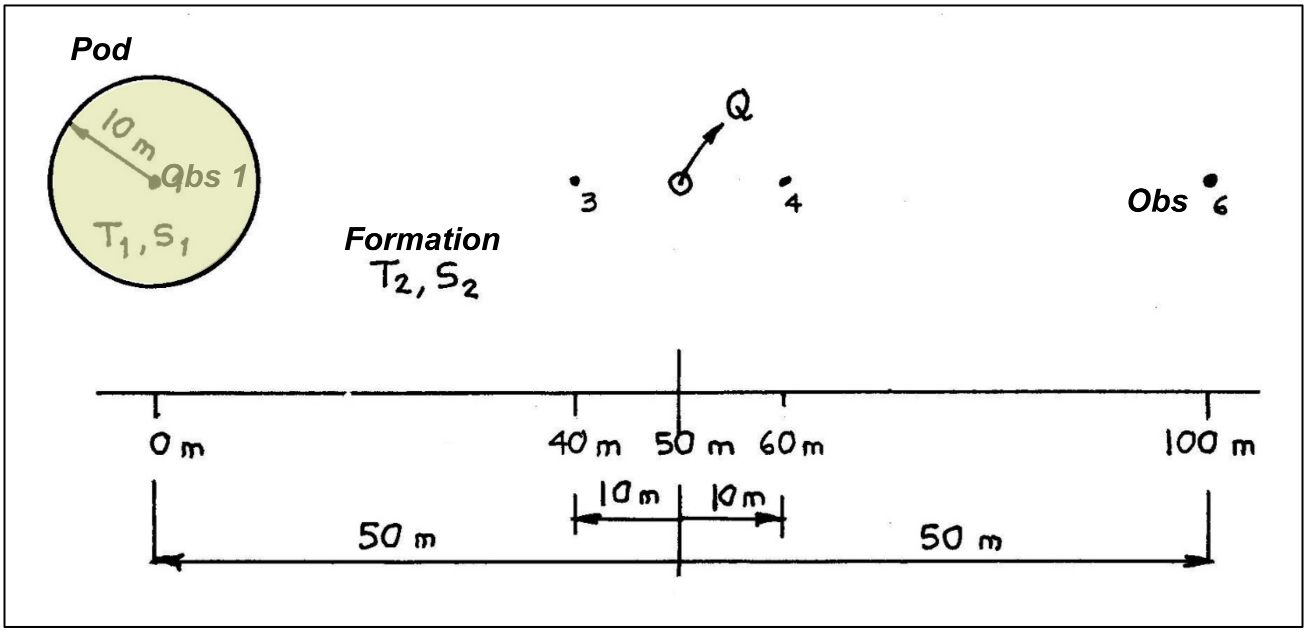

Figure 25. Conceptual model for Case #2 pod simulation

The simulated drawdowns for observation wells #3 and #4 are plotted in Figure 26. The drawdowns are essentially identical, which suggests that the observation wells 10 m from the pumping well are not affected by the presence of the pod.

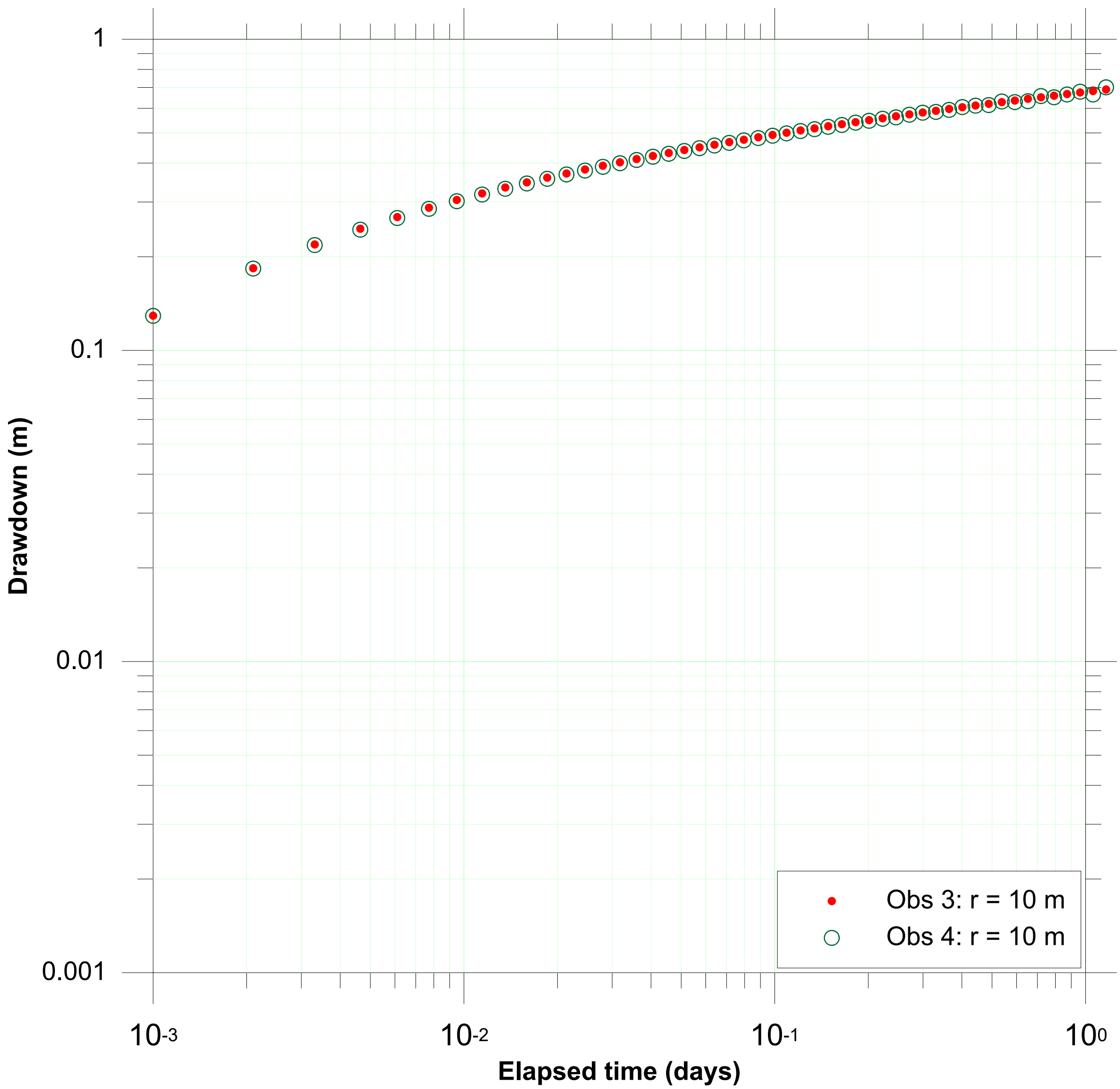

Figure 26. Simulated drawdowns at observation wells #3 and #4 (*r* = 10 m)

In Figure 27 the results of the Theis solution evaluated at r = 10 m with the formation properties, T_2 and S_2, are superimposed on the simulated drawdowns at observation wells #3 and #4. The Theis solution matches closely the simulated drawdowns.

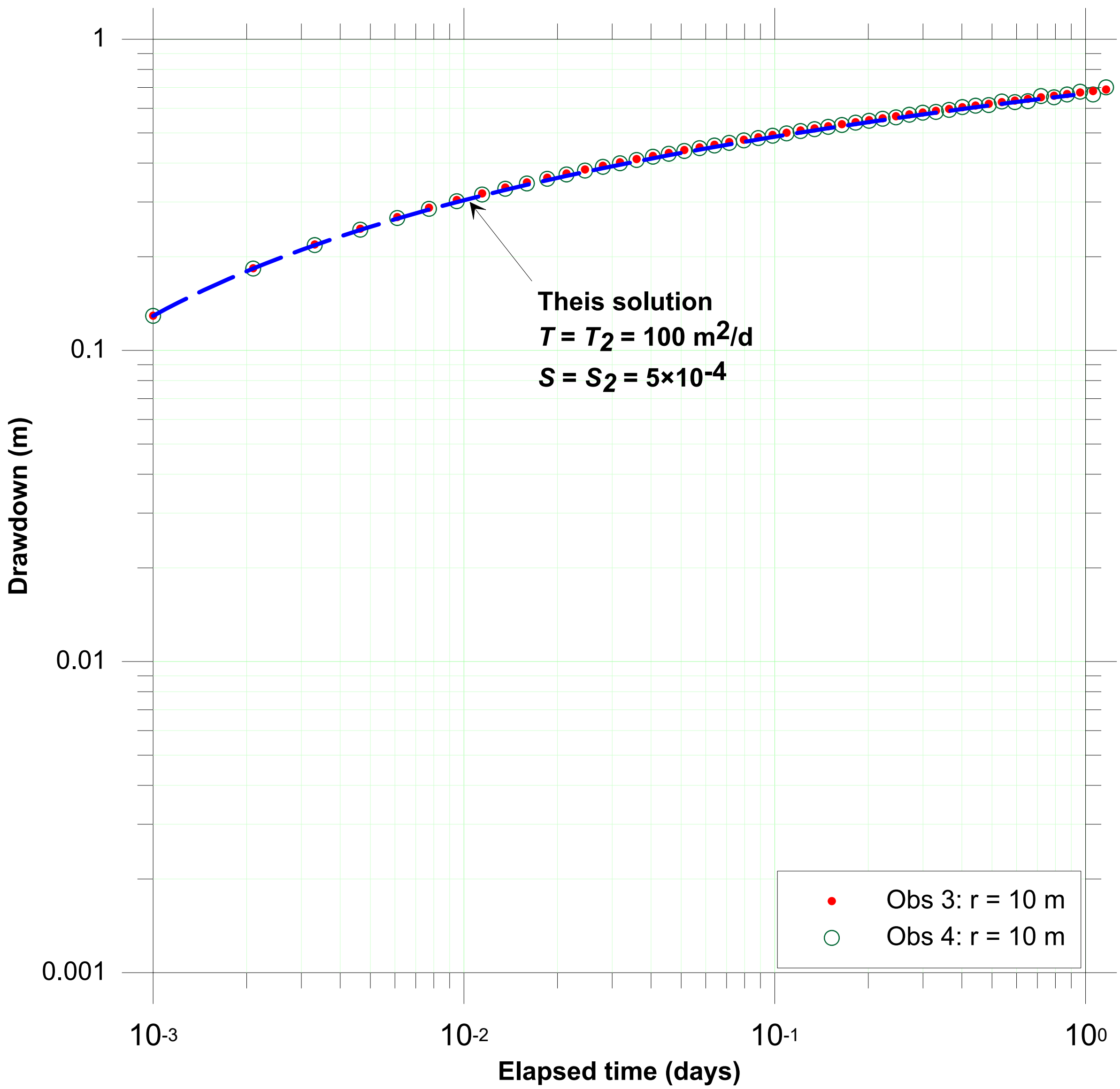

Figure 27. Match of the Theis solution to the drawdowns at observation wells at r = 10 m

The simulated drawdowns at wells located 50 m from the pumping well are plotted in Figure 28. The simulated responses for the two wells are quite different. These results provide a direct indication that the aquifer is heterogeneous.

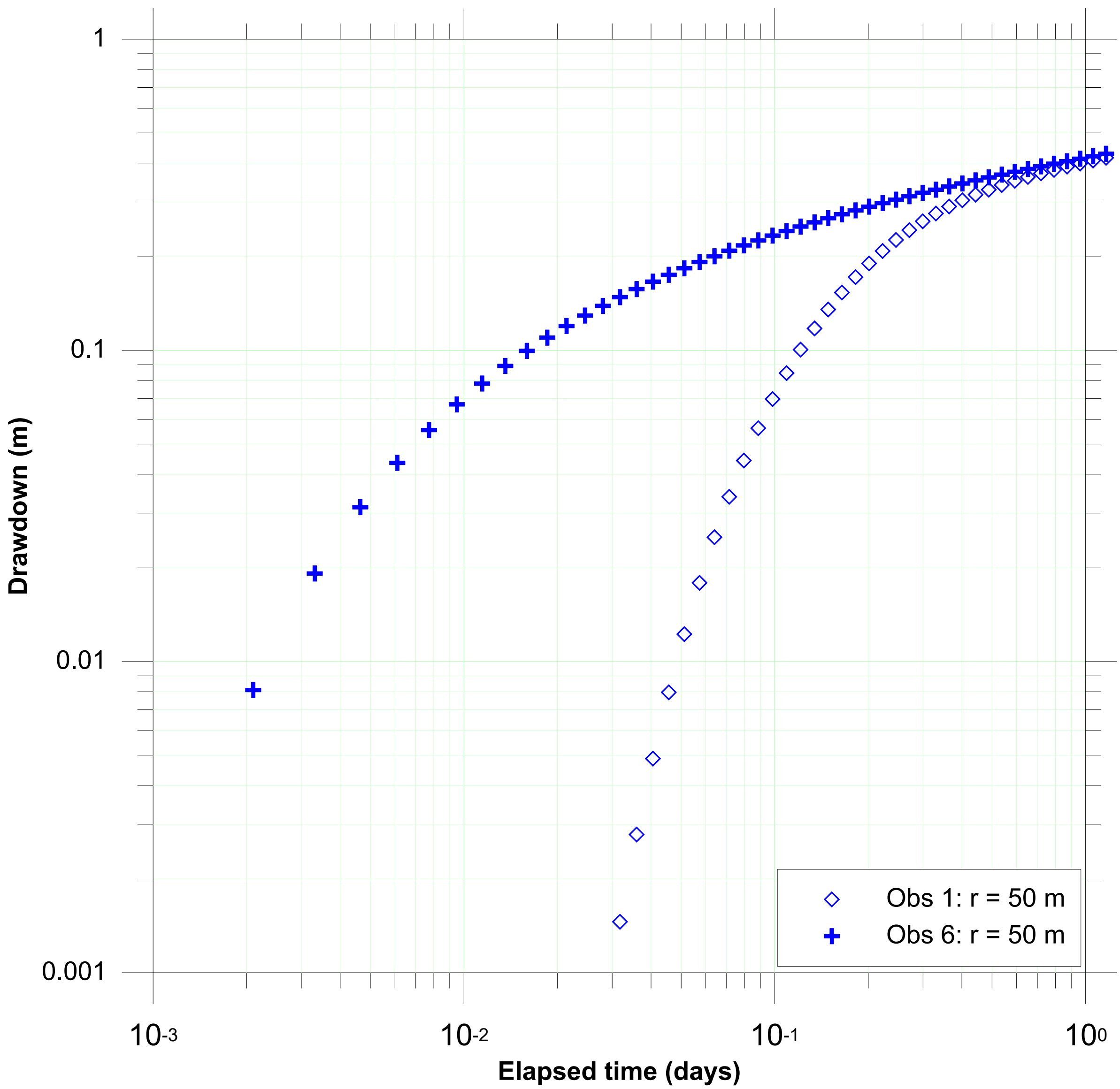

Figure 28. Simulated drawdowns at observation wells #1 and #6 (*r* = 50 m)

The match of the Theis solution to the drawdowns at observation well #6, located 50 m from the pumping well, is shown in Figure 29. The match yields the same parameter values as are specified for the formation.

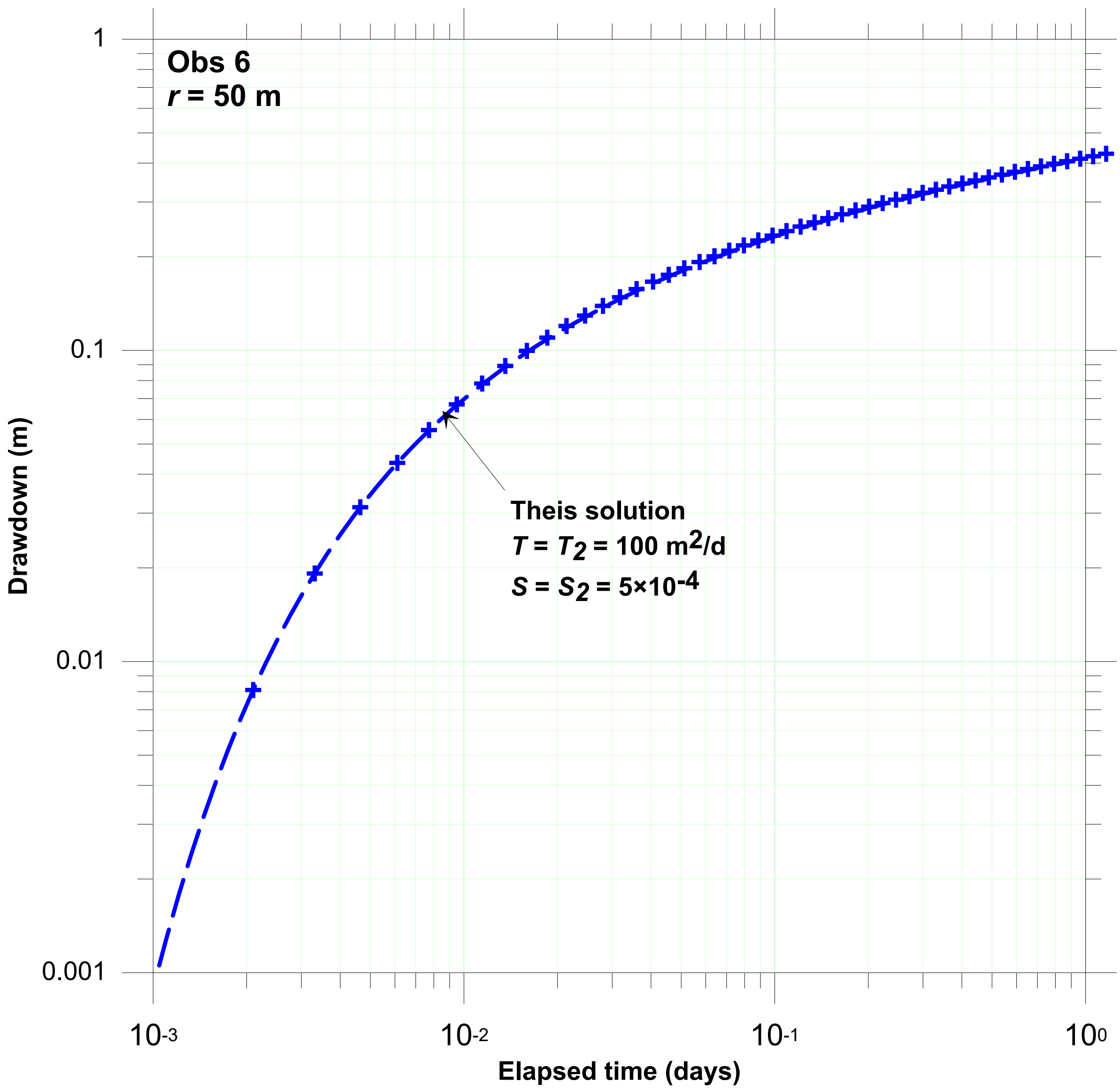

Figure 29. Theis analysis for observation well #6 (*r* = 50 m)

The results of a nonlinear regression match of the Theis solution to the drawdowns at well #1 are shown in Figure 30. In contrast to the results for well #6, it is not possible to achieve a good match to the complete record of simulated drawdowns with any combination of values of T and S. The best-fit analysis does not mimic either the early or later time trends of the drawdown record. The best-fit analysis yields a transmissivity of 42 m^2/d. Since the correct parameter values are already known, we can conclude that the transmissivity estimated for well #1 is not representative of either the formation or the pod in which it is located. Without the benefit of the correct parameter values, it would only be possible to note that something is amiss, as the fit is poor and the estimated transmissivity is significantly different than the value estimated for wells #3, 4, and 6.

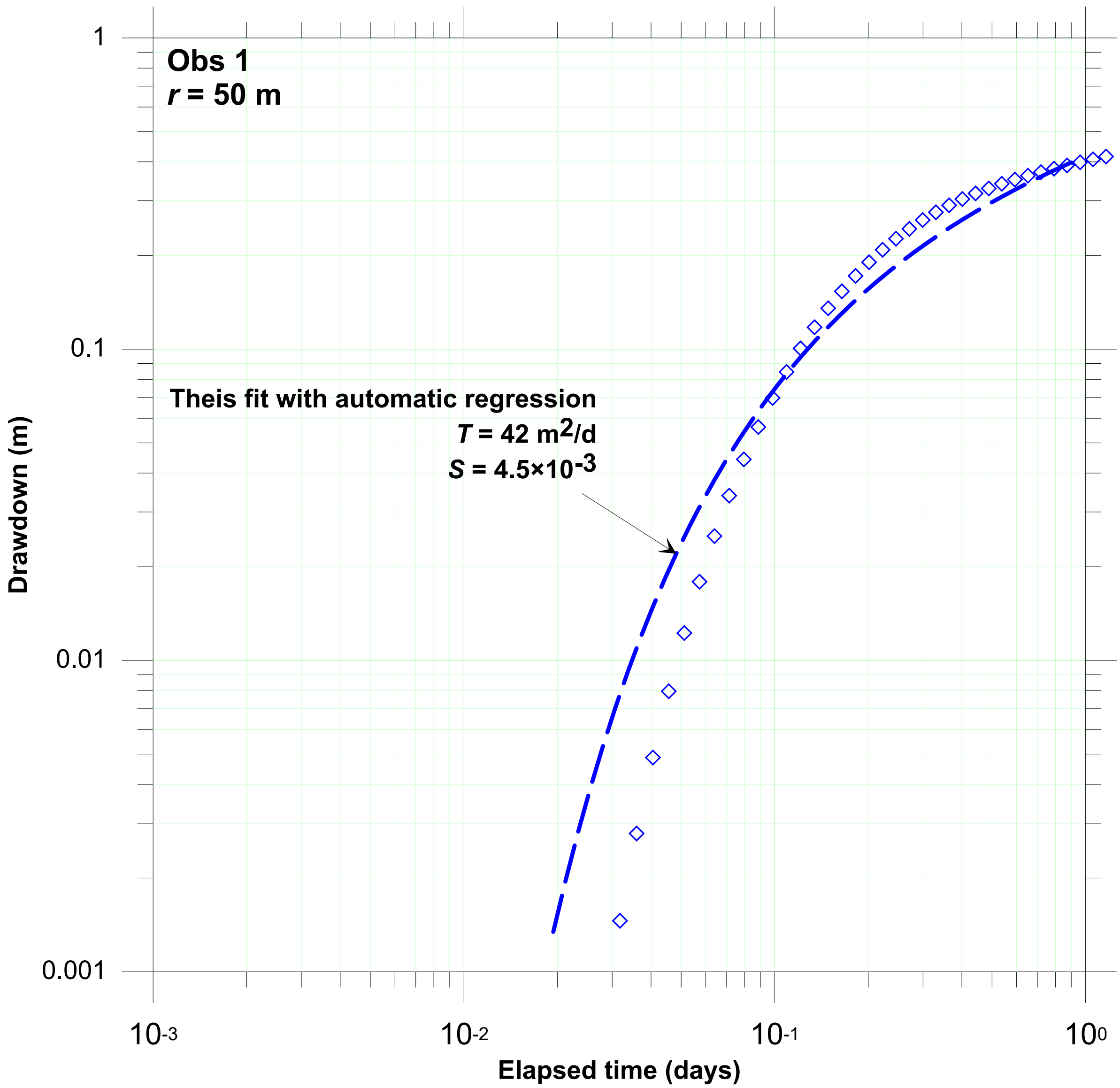

Figure 30. Theis analysis for observation well #1 (*r* = 50 m)

The results of the Theis solution evaluated 50 m from the pumping well with the formation properties, T_2 and S_2 are shown in Figure 31. It is unlikely that an analyst would be willing to accept the poor match to much of the drawdowns. If data similar to the simulated drawdowns shown in Figure 31 were obtained from an actual pumping test, and the properties of the formation were not known in advance, there is no guarantee that an analyst would recognize the correct portion of the response to match with the Theis solution.

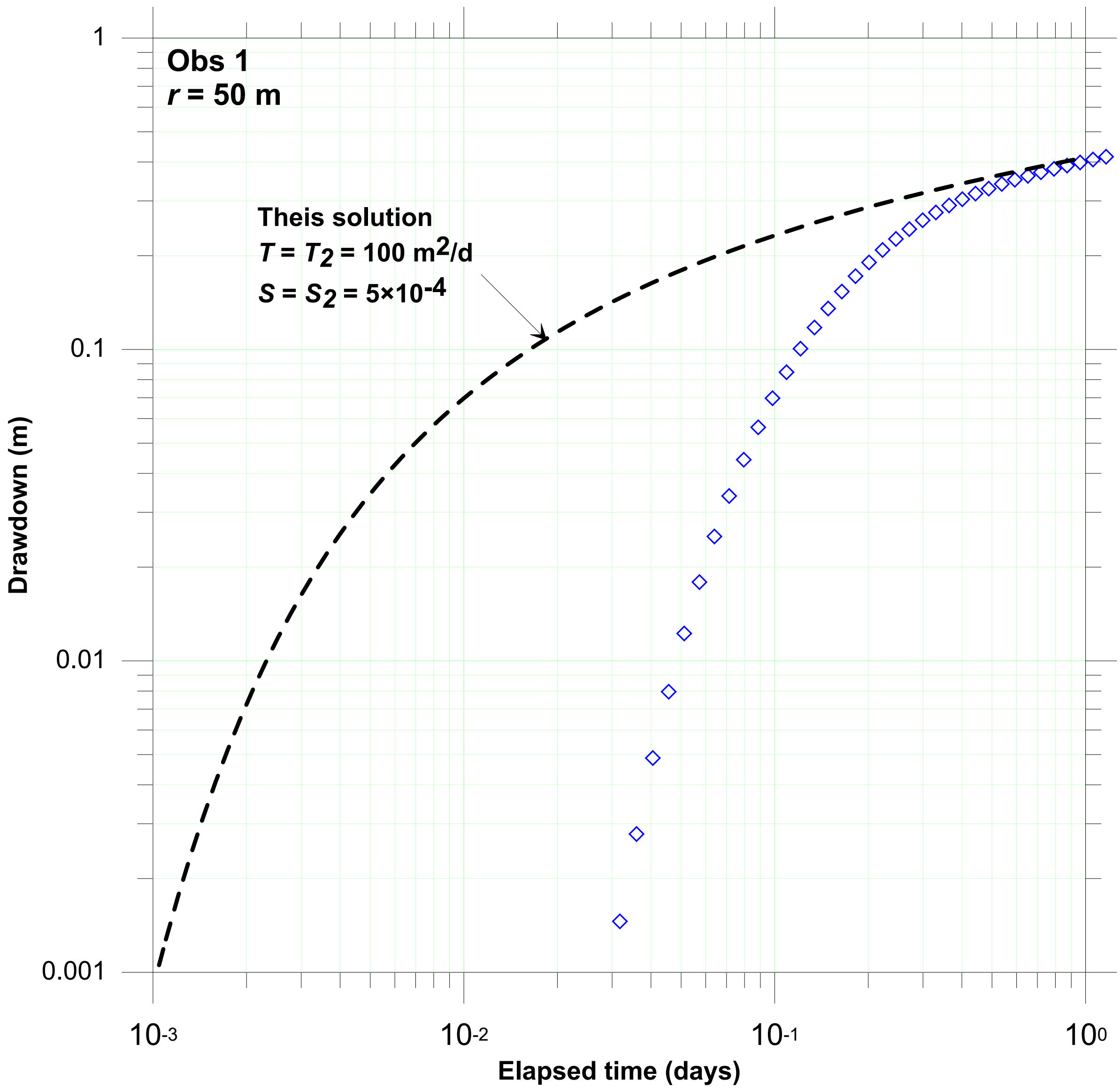

Figure 31. Theis solution with formation properties for observation well #1

The simulated drawdowns for the four observation wells are assembled on a semilog composite plot in Figure 32. In the figure, the drawdowns for three of the wells approximate the same line, while the early-time drawdowns for observation well #1 appear to be anomalous. The composite plot reveals that matching the early-time observations from well #1 is not appropriate.

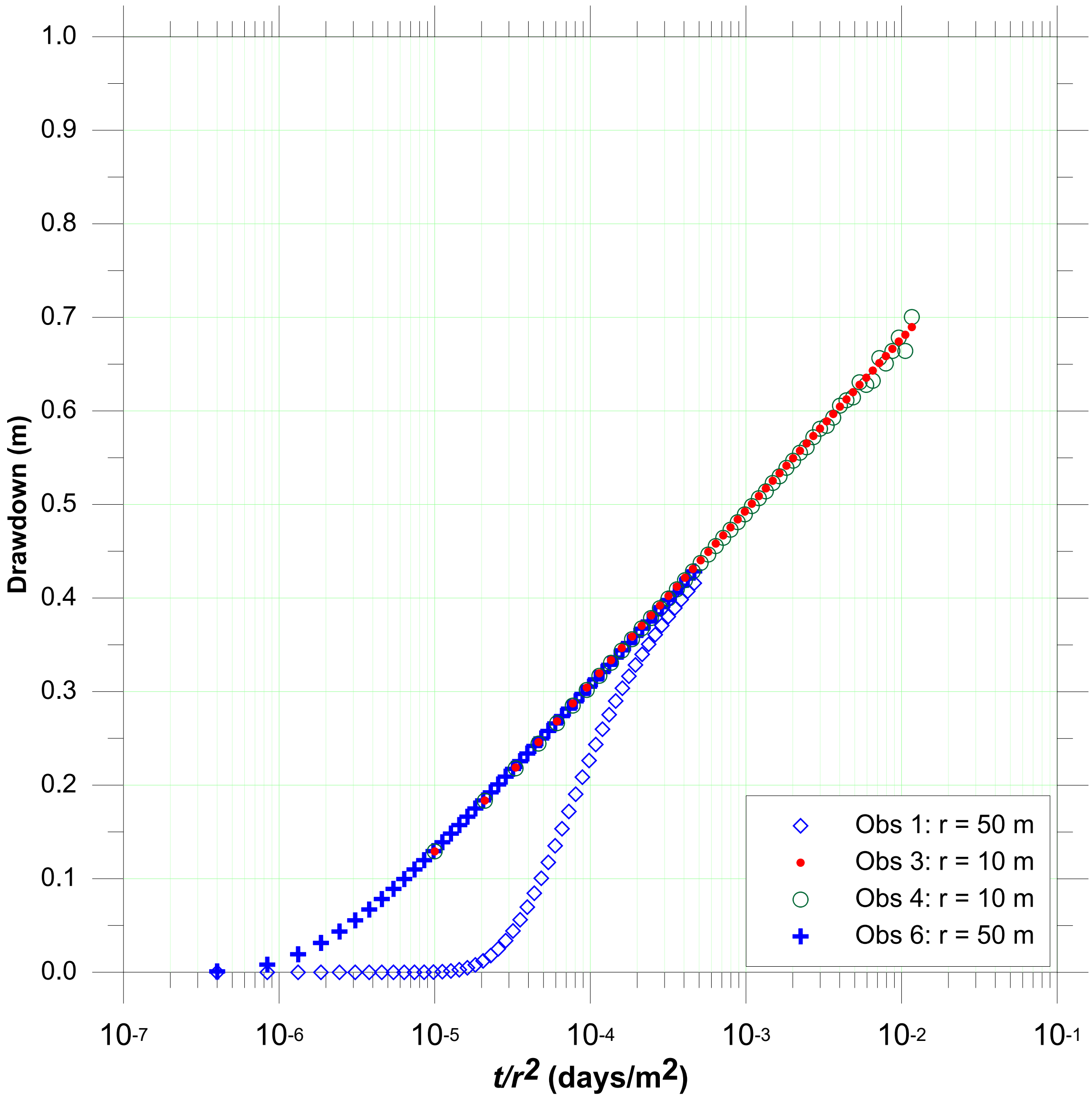

Figure 32. Semilog composite plot for Case #2 pod simulation

As shown in Figure 33, the simulated drawdowns for all four observation wells converge on the results predicted for a homogeneous aquifer with the properties of the formation. A Cooper-Jacob analysis conducted on the late-time data yields a transmissivity that is consisent with the value specified for the formation.

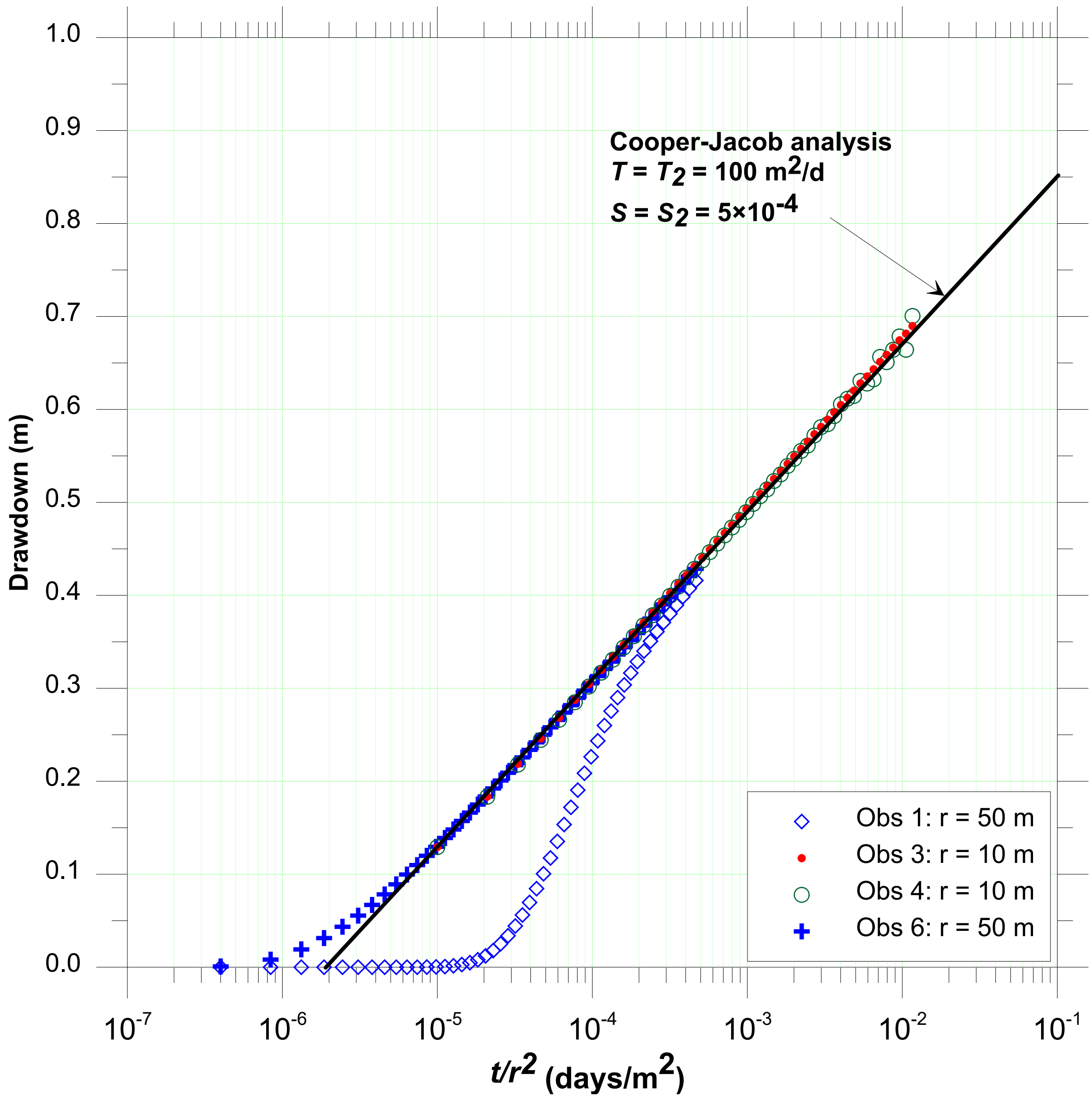

Figure 33. Semilog composite plot for Case #2 pod simulation with Theis solution for a uniform aquifer

Tentative conclusion from the "pod" experiments:

In aquifers that contain distinct zones it may be possible to take advantage of the strengths of the Cooper-Jacob composite analysis to identify the portion of the response that is representative of bulk-average radial flow, and to estimate a representative transmissivity from that portion of the data.

5. The significance of the composite plot

It is important to note that composite plots are not new. Cooper and Jacob (1946) refer to composite analyses in their classic paper, recommending that this plotting approach be adopted when time-drawdown records are available from multiple wells. This recommendation is echoed in the Stallman (1971) treatise on pumping test design:

> *Where several wells are available, predicted response should be plotted as t/r^2 or r^2/t for all wells on one sheet of graph paper.*

Weeks (1977) updated Stallman's report and included some incisive comments on composite plots. He wrote:

> *The composite data-curve matching process is also important during the analysis of the test data. Such a match should always be made when data from more than one observation well are available, and single values of transmissivity, storage coefficient, and other hydraulic properties are to be determined from that data.*

The results of numerical experiments of Meier and others (1998) for statistically homogeneous aquifers and simulations developed with the "pod" analytical solutions of Butler and Liu (1993) yield a consistent impression: it may be possible to estimate an effective transmissivity from a pumping test using the Theis model when applied with the semilog composite plotting approach.

When applied correctly with a focus on later-time data, a Cooper-Jacob analysis on a composite semilog plot analysis allows the analyst to look beyond the variability of the responses at individual observation wells. The composite plotting approach directs the analysts towards developing a single estimate of the transmissivity, consistent with the foundations of the analytical solutions typically used to interpret pumping test data.

A composite plot also has very useful diagnostic uses. When the data from a particular observation well do not plot on the same curve as the data from other wells, it is likely that the assumptions of the Theis solution are severely violated for this well. The observation well may be located in a pocket of material with properties that are significantly different from the portion of the formation from which the pumping well draws the bulk of its supply. Alternatively, the observation well may not be located in the same aquifer as the pumped well.

Cooper and Jacob (1946; page 534) make a pithy comment that highlights the value of composite plots as an aid to diagnosing aquifer response:

> *The extent to which these or other circumstances might vitiate the method used may be judged most readily from the alignment of the points on a simple, straight-line graph* [a composite plot].

Allan Moench, a giant of contemporary aquifer test interpretation recently retired from the United States Geological Survey, advocated the use of composite analyses in 1994. The specific mention of composite analyses is (Moench, 1994; page 950):

> *"This procedure* [plotting drawdown data against the composite t/r^2 axis] *is important for proper interpretation of the response of the aquifer as a whole and in obtaining better "average" aquifer properties. It can and it will be demonstrated, in fact, that one can be seriously misled by attempting to analyze data from a single point of observation."*

In a recent paper in *Hydrogeology Journal*, Yeh and Huang (2009) attempted to demonstrate that parameter estimates obtained from composite analyses are no better, and perhaps even worse than parameter estimates obtained from the analyses of individual time-drawdown records. We thought that the paper was nonsense, and Moench (2010) has written a discussion to the paper that confirms our impression. Moench concludes his discussion by noting that because it can simplify analysis with the simultaneous use of all available drawdown data, composite analysis is to be preferred for regional aquifer-test analysis. He concludes that use of composite analysis is an essential element for analysis as it allows for input from an experienced hydrogeologist to account for non-ideal aquifer conditions.

We argue that since the underlying analyses assume that the aquifer is homogeneous, a key objective of the analysis must be to estimate single values that are representative of average properties. Estimating multiple values proves only that the assumptions underlying the analysis have been violated.

Our review of Standard Operating Procedures (SOPs) promulgated by government agencies suggests that there are generally no mentions or advocacy for composite analyses. Moench indicates that although the recommendation to plot data on a composite plot is important it is often disregarded (Moench, 1984; page 950). However, it is indicated in Section 8 of ASTM Standard D4106-96 (Theis analysis) that the drawdown data should be plotted against time if one observation well is used, and against t/r^2 if more than one observation well is used.

6. Summary of key points

1. The Cooper-Jacob method is the simplest method of interpretation in our toolkit. This simplicity can be deceptive: the method frequently yields the most reliable estimates of transmissivity. There seems to be little appreciation of its underlying strengths.

2. The results of numerical experiments of Meier and others (1998) suggest that for homogeneously heterogeneous aquifers it may be possible to estimate an effective transmissivity from a pumping test using the Theis model. The results of recent stochastic simulations support the earlier conclusion of Tóth (1966):

   "…the complexity of the actual (geological) situation can not be described in a rational way. The strata are <u>not</u>: isotropic, homogeneous, of infinite area extent, wholly confined or completely free …. Yet, the end results of the Theis concept seem to be quite satisfactory."

3. In our opinion, the most reliable interpretations of aquifer tests in confined aquifers are accomplished with a composite plot, following the approach suggested in the original paper of Cooper and Jacob (1946). This approach has three important advantages. First, it assists in identifying those responses that are significantly different, that is, the outliers. If the data from one observation well do not plot on the same line as other data, the assumptions of the Theis solution are severely violated for this well. The observation well may not be located in the pumped aquifer, or the observation well may be located in a pocket of material with significantly different properties. Second, it directs the analysts towards estimating a single, bulk average, estimate of the transmissivity. Third, the semilog plot tends to emphasize later time data.

7. References

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Pumping test case study: NDPW1-08 (Cambridge, Ontario)

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Last update: April 28, 2025

1. Introduction

Well NDPW1-08 was installed and tested as part of a program to investigate additional municipal groundwater supplies for the city of Cambridge, Ontario (Figure 1).

Figure 1. Location map

NDPW1-08 is cased across the overburden and constructed as a 12-inch diameter open hole in the bedrock (r_w = 0.15 m). The primary production intervals for NDPW1-08 are flow zones in the dolomitic rocks of the Gasport Formation.

The testing of NDPW1-08 included the execution of a step test, followed by a constant-rate pumping test. In this case study the drawdowns observed during the constant-rate pumping test are analyzed, with analyses that evolve in their complexity. The final objective of the analyses is to match all of the data with a conceptual model that is internally consistent.

2. Data collected during the constant-rate pumping test

During the constant-rate pumping test NDPW1-08 was pumped for 6 days at an average rate of 50 L/s (4,320 m^3/d).

The drawdowns were recorded at the pumping well and at 9 observation wells (Figure 2). The distances between the pumping well and the observation wells are listed below. The time-drawdown records are plotted in Figure 3.

Well	Distance from NDPW1-08, *r* (m)
NDPW1-08	0.15
NDTW2A-08	3.54
NDTW1A-08	156.79
NDOW1A-08	664.43
NDOW2A-08	1042.13
CMOW1A-06	3707.77
CMOW2A-06	2631.96
CMPW2-06	3274.02
PBOW1A-06	3542.39
SMTW1A-05	3720.39

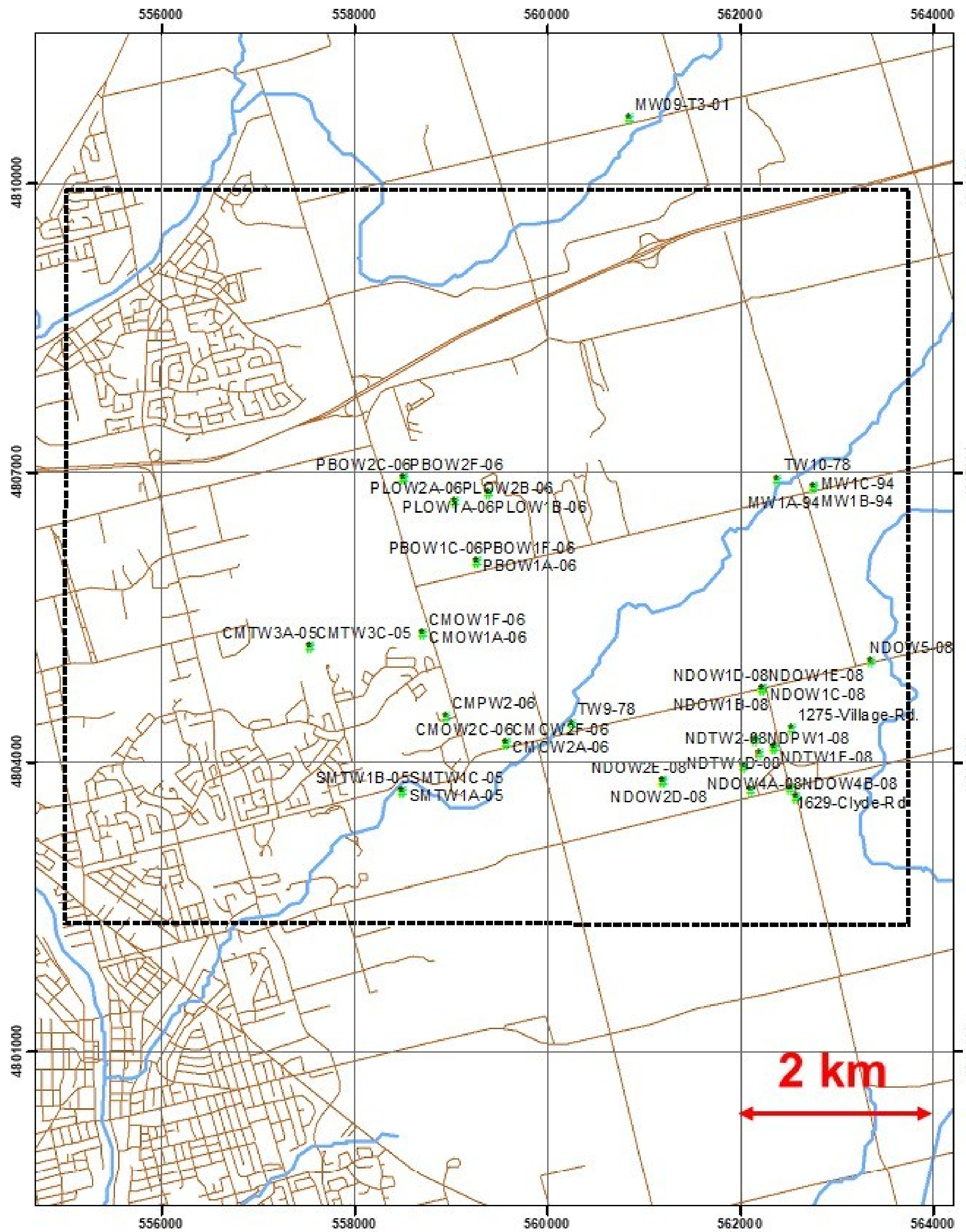

Figure 2. Locations of observation wells for the constant-rate pumping test

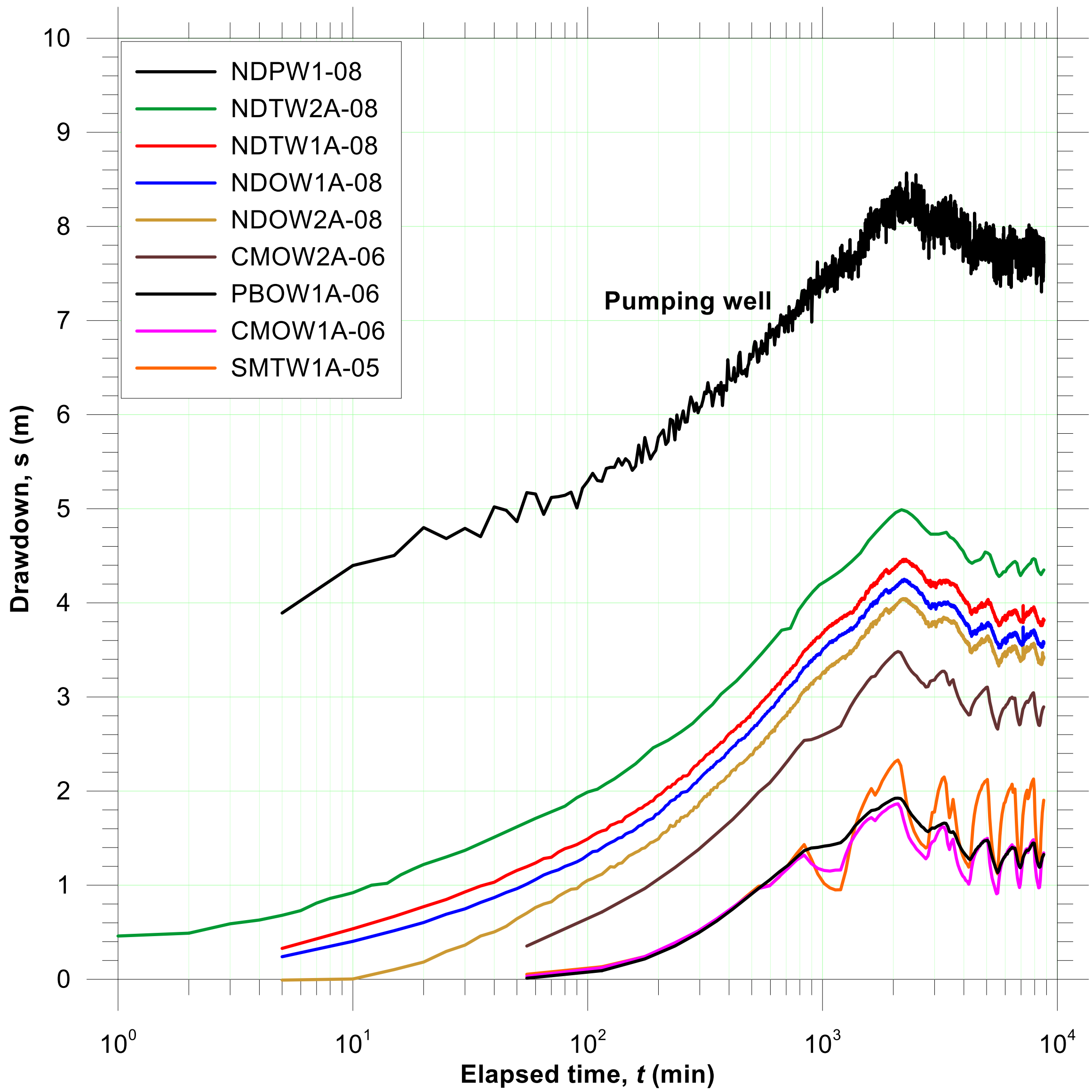

Figure 3. Drawdowns during the NDPW1-08 constant-rate pumping test

3. Adjustment of the drawdowns in the pumping well

When data from a step test are available, it is possible to adjust the observed drawdowns for the pumping well so that these data may be treated as another observation well. The adjustment consists of removing the component of the total drawdown that is attributed to nonlinear well losses. A nonlinear well loss coefficient, $C = 0.303$ m/(m^3/min)^2 has been estimated from the step test data, which corresponds to 2.727 m of drawdown. The time-drawdown records are re-plotted in Figure 4.

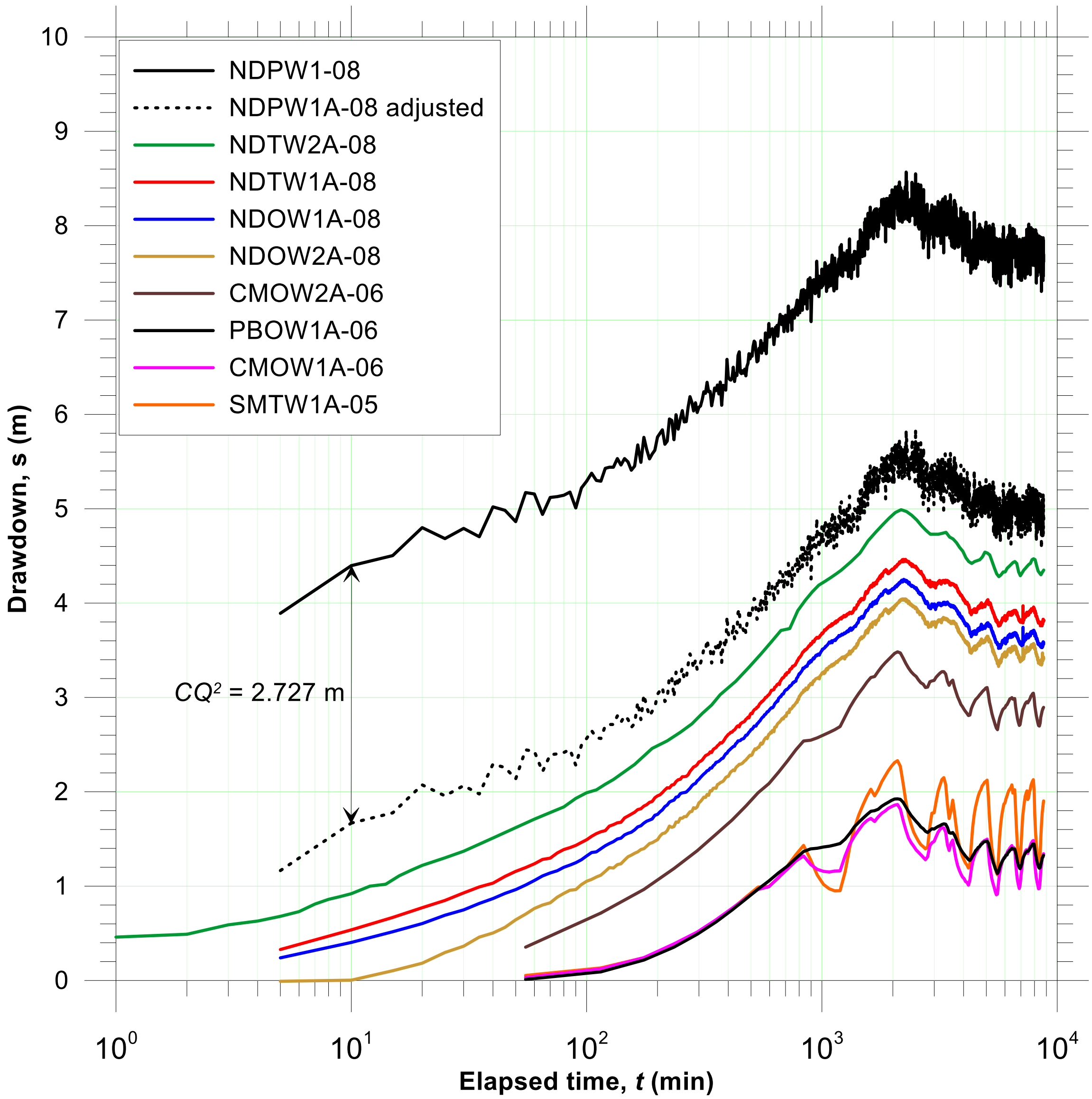

Figure 4. Drawdowns during the NDPW1-08 constant-rate pumping test, with corrected pumping well drawdowns

4. Initial composite plot for the constant-rate pumping test

When drawdown data are available for multiple wells, the appropriate presentation of the data is on a composite plot. For each well, the drawdown observed at time t is plotted against t/r^2, where r is the distance between the pumping well and the observation well. The semilog composite plot is shown in Figure 5.

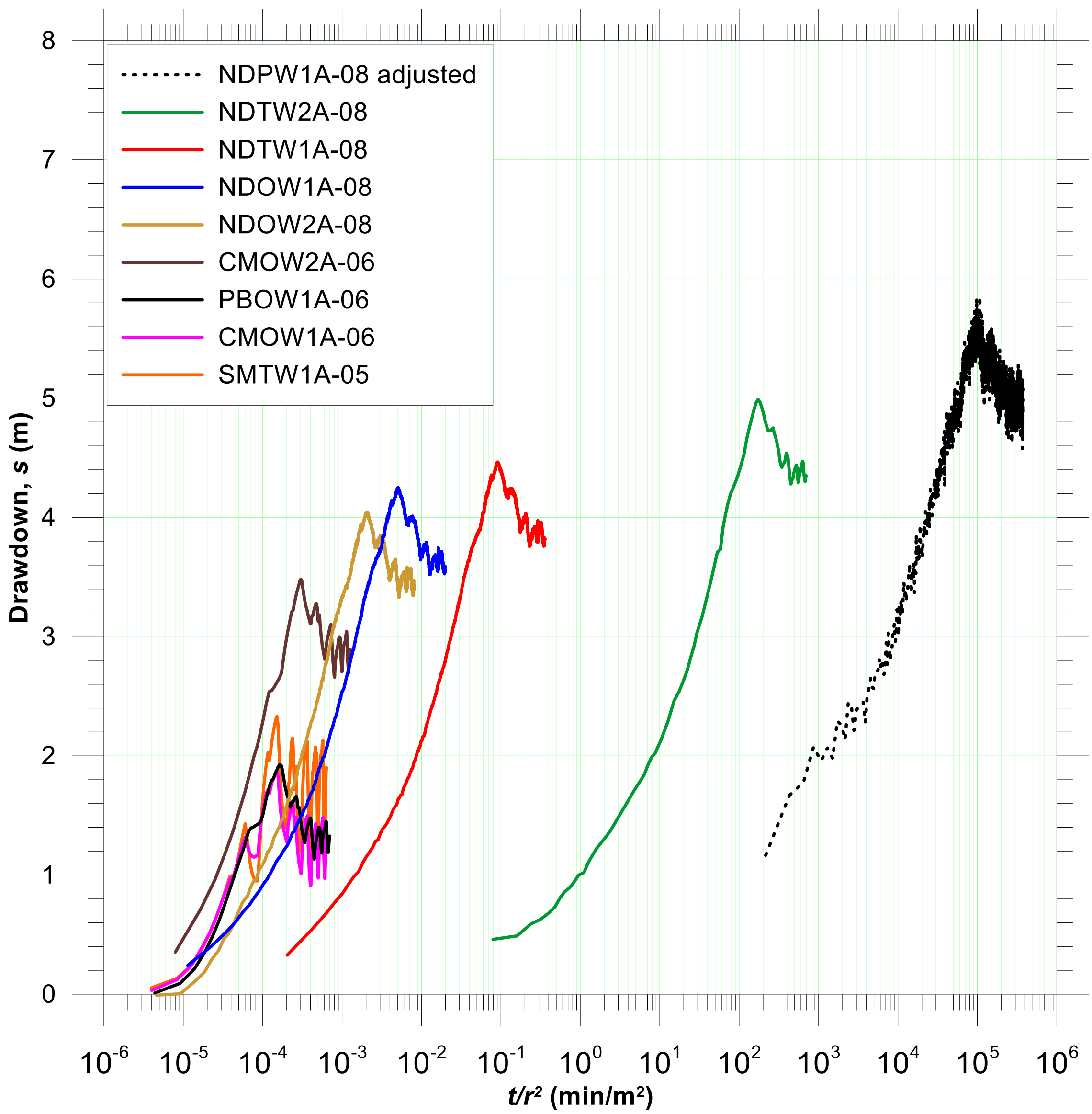

Figure 5. Composite plot for the constant-rate pumping test

The data presented in Figures and 5 suggest that it is straightforward to obtain a consistent estimate of the bulk-average transmissivity, as all of the observation wells have similar semilog slopes. This is illustrated in Figure 6, in which we have superimposed straight lines with identical slopes on individual records. In this case, the slope is 2.6 m per log cycle t/r^2, which yields a transmissivity of **305 m²/d**.

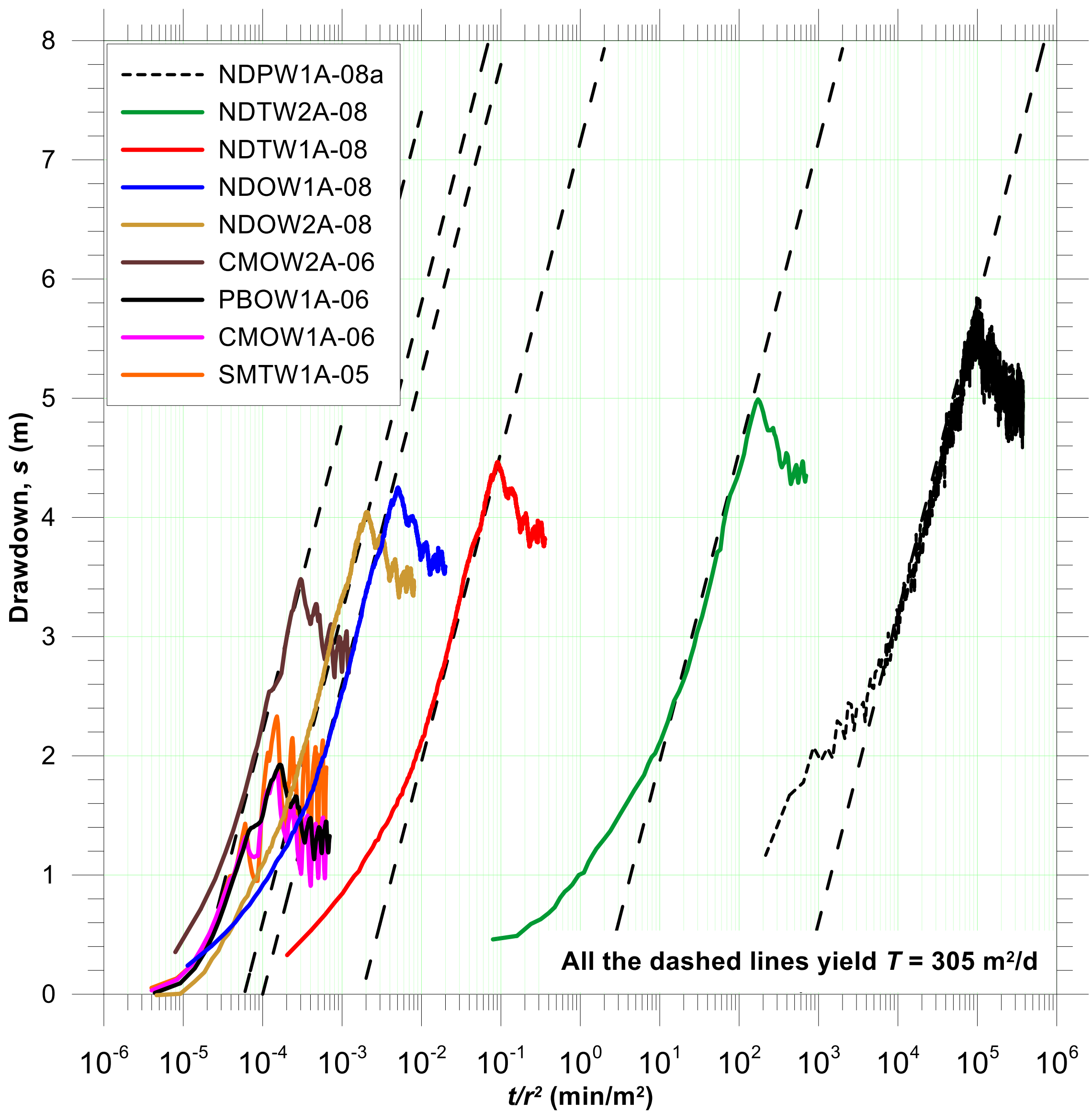

Figure 6. Composite plot with Cooper-Jacob straight-line analysis

5. Assessment of the initial Cooper-Jacob straight-line analysis

If our only objective was to obtain a consistent estimate of the bulk-average transmissivity, we could stop with the analysis of Figure 6. However, we also want to understand why the individual wells respond as they do. In this respect, the drawdown records plotted in Figure 5 are puzzling. For an ideal confined aquifer, the drawdowns for all of the wells should fall on a single straight line. As shown in Figure 6, there are almost as many straight lines as there are observation wells. In this case, the Cooper-Jacob analyses yield consistent estimates of transmissivity, but with estimates of the storage coefficient that vary over a wide range. The variation of the storage coefficients suggests that the structure of the subsurface is significantly more complex than conceived with the Theis model.

To gain more insight into the data set, the drawdown records for the individual wells are revisited in Figure 7. Two aspects of the drawdown data are noteworthy. First, the drawdowns are relatively smooth up to about 2000 minutes; the drawdowns decline beyond this time and follow oscillating patterns. The pumping rate was held constant during the test, so the irregularities are not due to pumping from NDPW1-08. They are likely due to the influence of nearby municipal production wells, for example, G16. Second, the responses of all of the wells appear to track each other closely, both during the "smooth" period and the later period of irregular response.

The wells appear to fall into two general groups:

- Group #1: The pumping well and observation wells NDTW2A-08, NDTW1A-08, NDOW1A-08 and NDOW2A-08; and

- Group #2: Observation wells PBOW1A-06, CMOW1A-06, and SMTW1A-05. These wells are each more than 3000 m from the pumping well. The irregular responses begin earlier for these wells, which is consistent with their relatively close proximity to G16.

Well CMOW2A-06 appears to respond as if it were in a transition zone between these two groups.

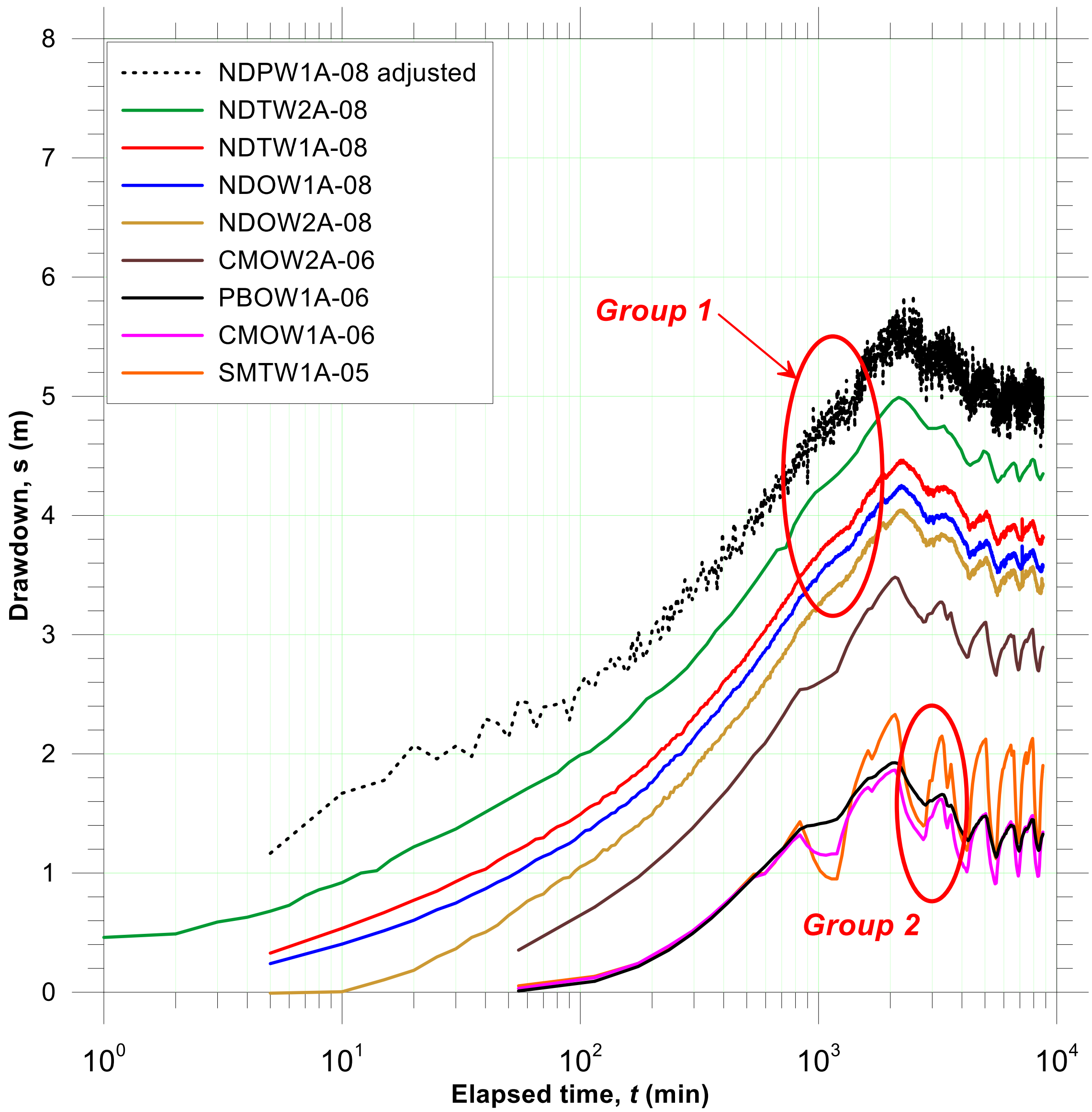

Figure 7. Drawdown versus time for the pumping and observation wells

6. Distance-drawdown analysis

In a second attempt to gain more insight into the data set, in Figure 8 the maximum drawdown for each observation well are plotted against its distance from the pumping well. The maximum drawdowns are observed about 2,000 minutes after the start of the constant-rate pumping test. As shown in Figure 8, the drawdowns appear to approximate two straight lines. The wells on the first straight line belong to Group #1, and the wells on the second straight line belong to Group #2.

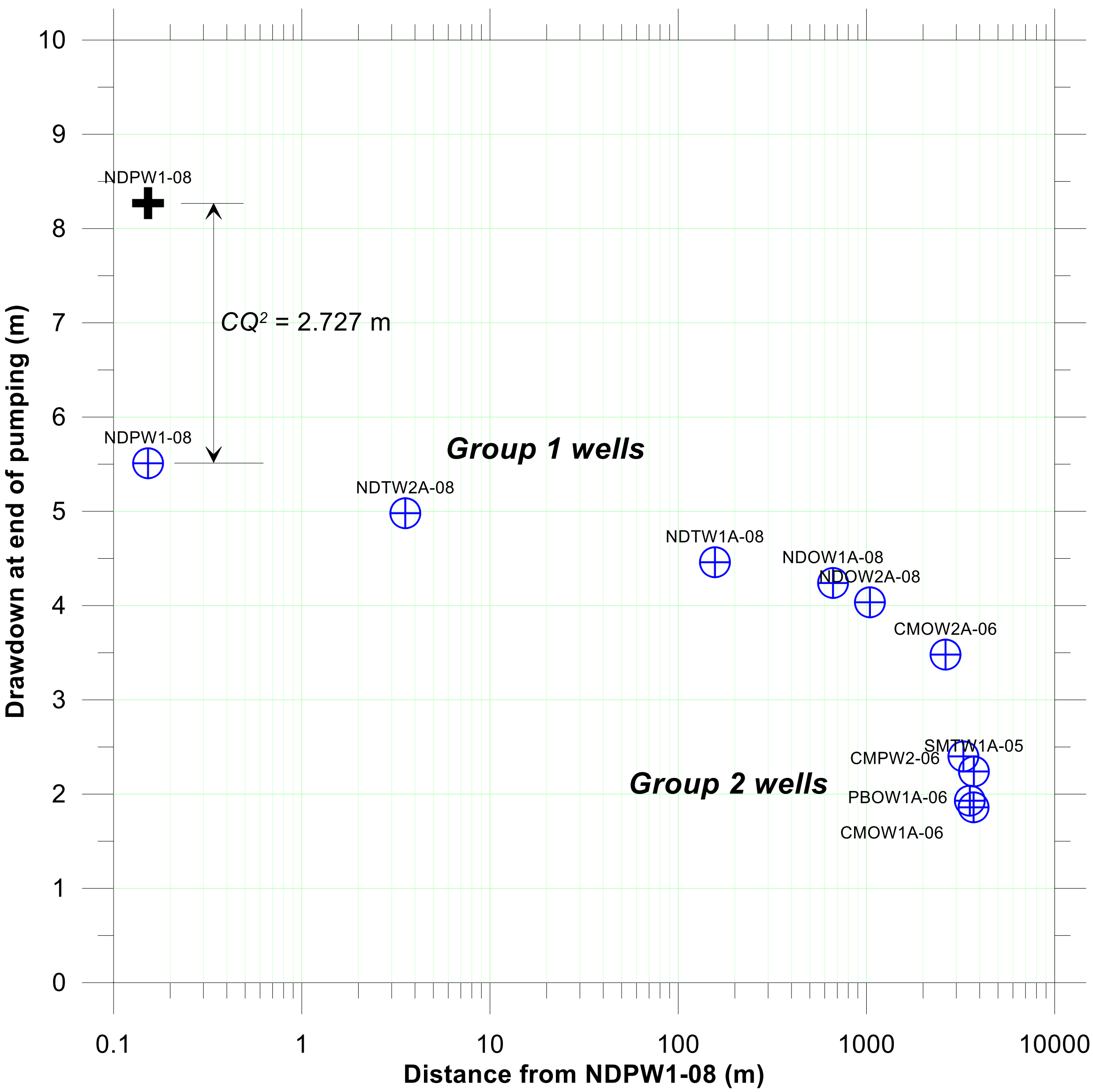

Figure 8. Maximum drawdown versus distance from the pumping well

The two slopes on the distance-drawdown plot are shown in Figure 9. The corresponding transmissivities estimated with Cooper-Jacob straight-line analyses are:

- Group 1 wells ($SLOPE_1$): $T = 4540$ m^2/d; and
- Group 2 wells ($SLOPE_2$): $T = 305$ m^2/d.

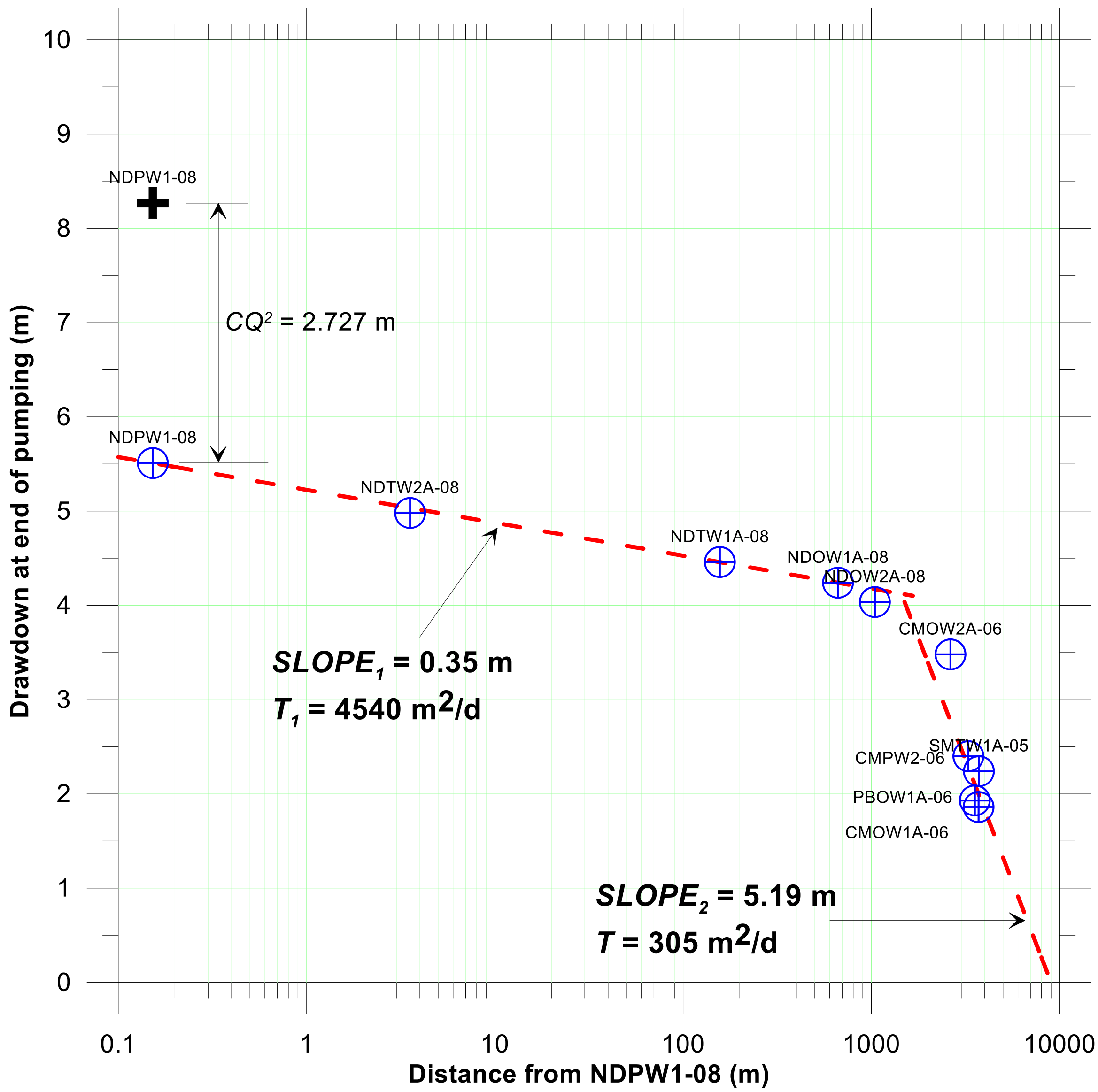

Figure 9. Cooper-Jacob distance-drawdown analyses

7. Analysis to match all of the data: Interpretation with a slightly more complex conceptual model

The results of the distance-drawdown analysis are used as the starting point for a complete analysis with a more complex conceptual model. The conceptual model of Barker and Herbert (1982) is shown schematically in Figure 10. All of the assumptions of the Theis model are invoked with the exception of one: the aquifer is assumed to consist of two distinct zones. The pumping well is assumed to be located at the center of an inner zone, which is surrounded by a zone of uniform properties corresponding to the bulk formation. [Correct solutions for this model have also been presented by Loucks and Guerrero, (1961), Sternberg (1969) and Butler (1988).]

Inner zone: $r < R$, $T = T_1$, $S = S_1$
Outer zone (formation): $r > R$, $T = T_2$, $S = S_2$

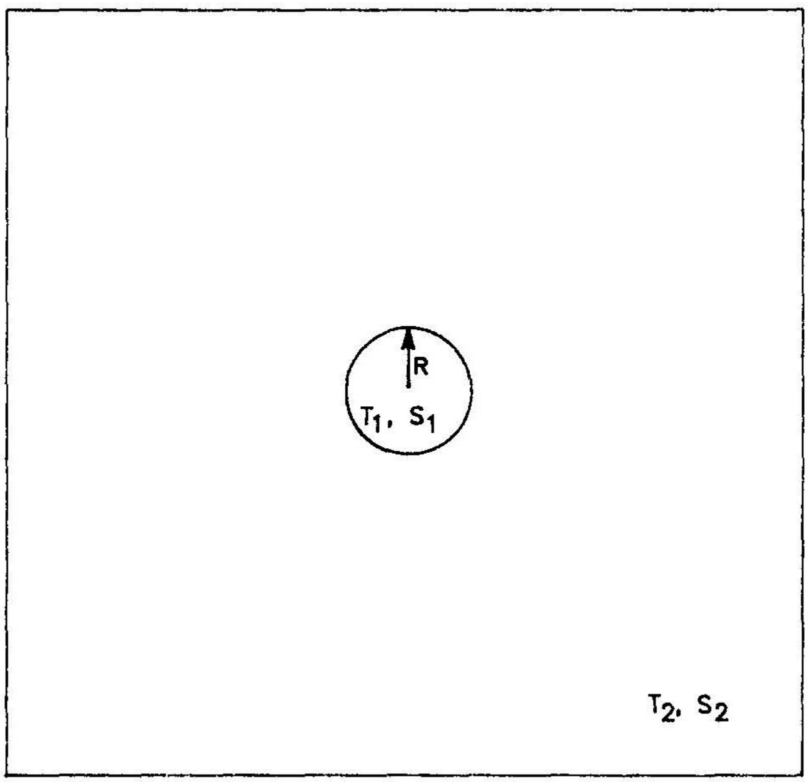

Figure 10. Conceptual model for the Barker and Herbert (1982) solution

For simplicity, it is assumed that the storage coefficients S_1 and S_2 are both 10^{-5}. Referring to the distance-drawdown plot in Figure 9, the zone around the pumping well is specified to extend for a radial distance of 1460 m. The transmissivity values estimated with the Cooper-Jacob distance-drawdown analysis are specified for the two zones, T_1 = 4540 m^2/d, and T_2 = 305 m^2/d. The results of the Barker-Herbert solution are plotted in Figure 11. As shown in the figure, an excellent match is obtained to the final drawdowns.

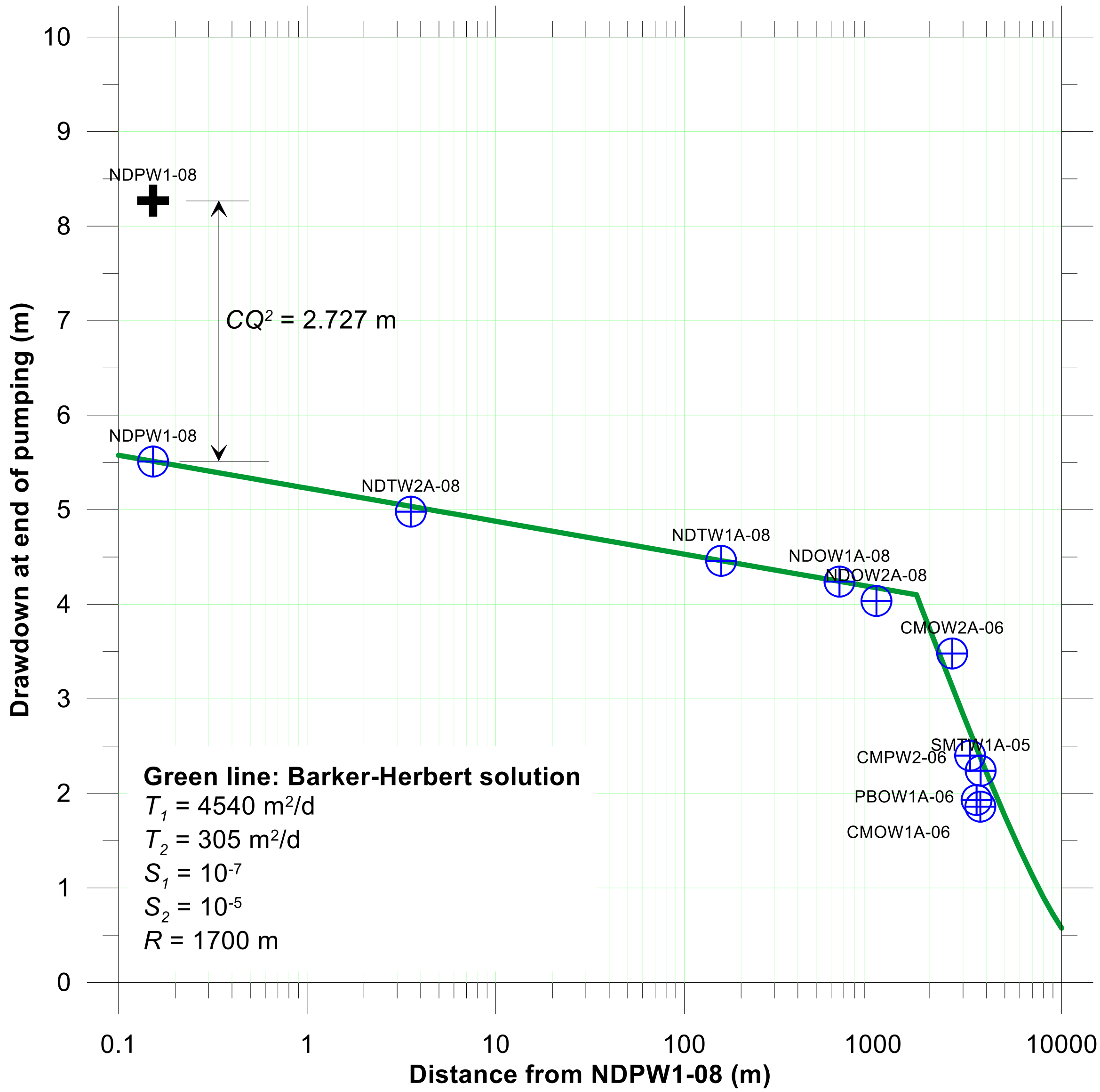

Figure 11. Match to drawdowns with analytical solution of Barker and Herbert (1982)

8. Composite plot with the Barker and Herbert (1982) model

The match to the final drawdowns presented in Figure 11 is encouraging. But what do the matches to the complete time-drawdown records look like? Specifying the parameters listed in Figure 11, complete records for selected wells calculated with the Barker-Herbert model. For simplicity, the simulated results for only four of the wells are plotted in Figure 12: the pumping well (adjusted drawdowns), NDTW2A-08 at 3.54 m, NDTW1A-08 at 156.7 m, and SMTW1A-05 at 3720 m. The match to the complete sets of observations is excellent.

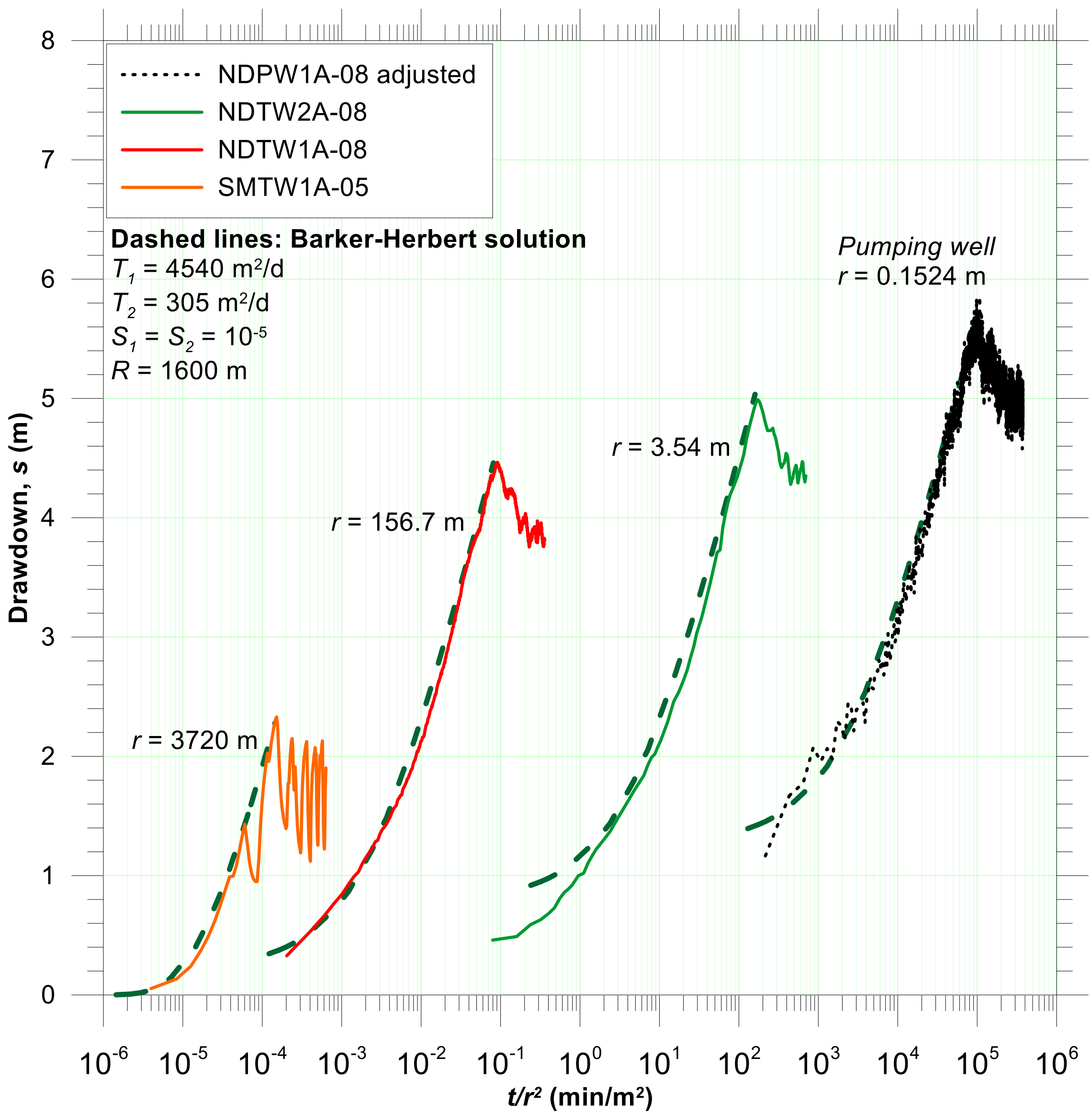

Figure 12. Barker-Herbert solution, composite plot

9. Sensitivity analysis with the Barker and Herbert (1982) model

To assess the sensitivity of the calculated drawdowns with respect to the parameters of the Barker and Herbert (1982) model, the following variations of the key parameters are examined:

- Extent of the inner zone, R: 2 × base case, 0.5 × base case;
- Transmissivity of the inner zone, T_1: 2 × base case, 0.5 × base case; and
- Transmissivity of the outer zone, T_2: 2 × base case, 0.5 × base case.

Sensitivity Run	R (m)	T_1 (m^2/d)	T_2 (m^2/d)
Base case	1600	4540	305
1	800	4540	305
2	3200	4540	305
3	1600	9080	305
4	1600	2270	305
5	1600	4540	610
6	1600	4540	152.5

The results presented in Figures 13, 14 and 15 suggest that these parameters affect the drawdowns in different ways. Therefore, the available data are sufficient to constrain the values of R, T_1, and T_2.

a. <u>Sensitivity with respect to the extent of the inner zone, *R*</u>

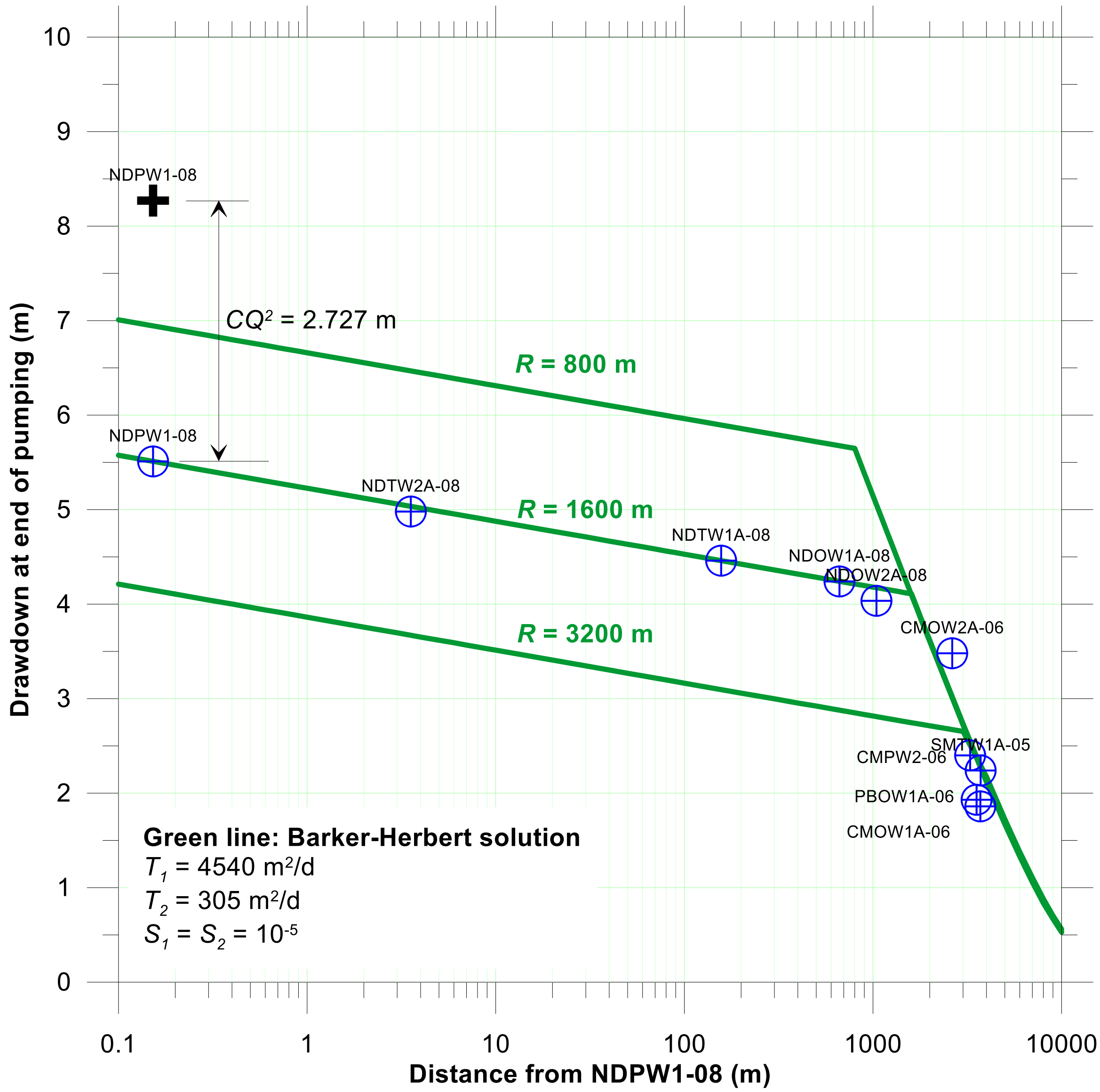

Figure 13. Barker-Herbert model, sensitivity analysis with respect to *R*

b. Sensitivity with respect to the transmissivity of the inner zone, T_1

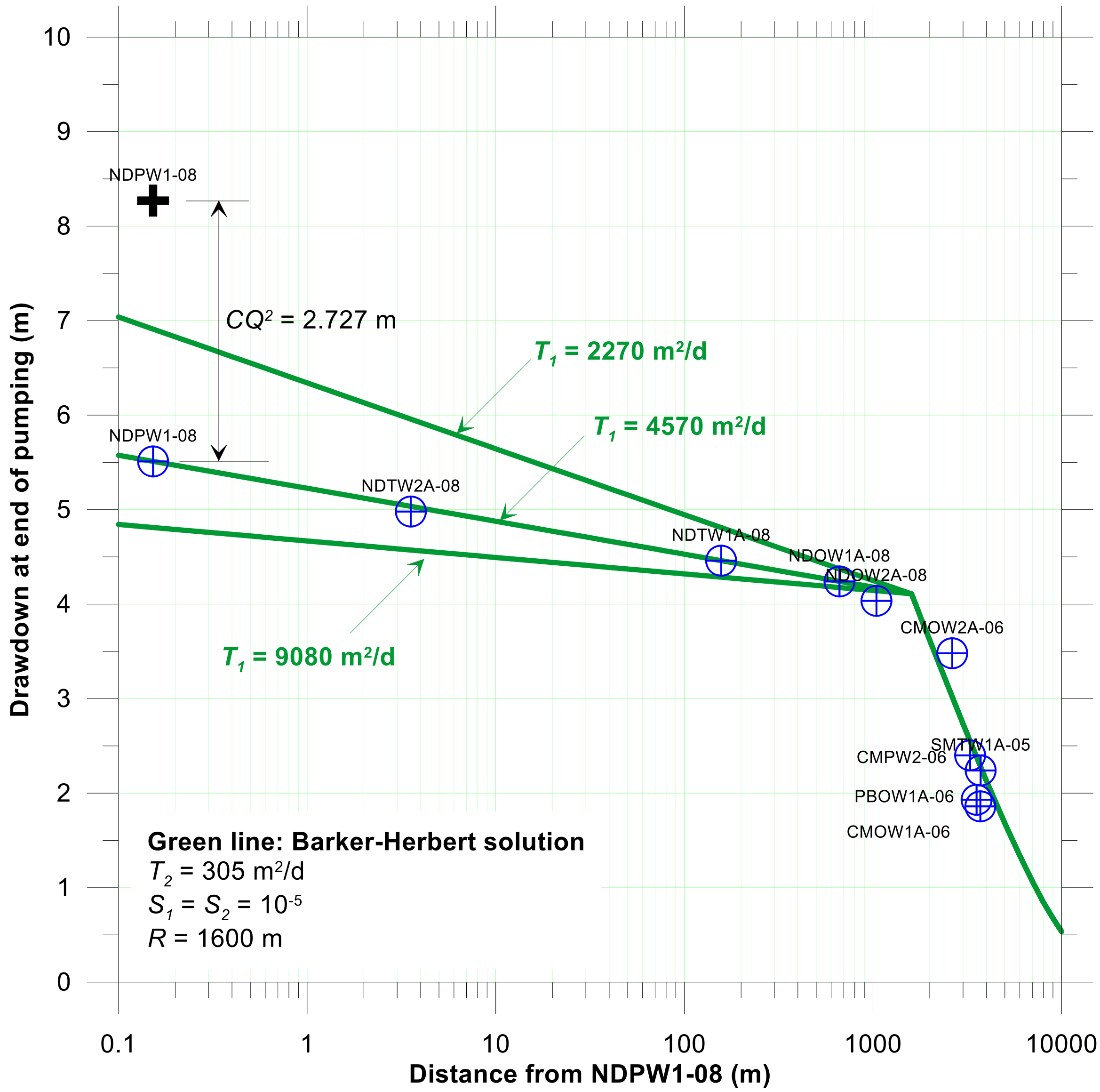

Figure 14. Barker-Herbert model, sensitivity analysis with respect to T_1

c. <u>Sensitivity with respect to the transmissivity of the outer zone, T_2</u>

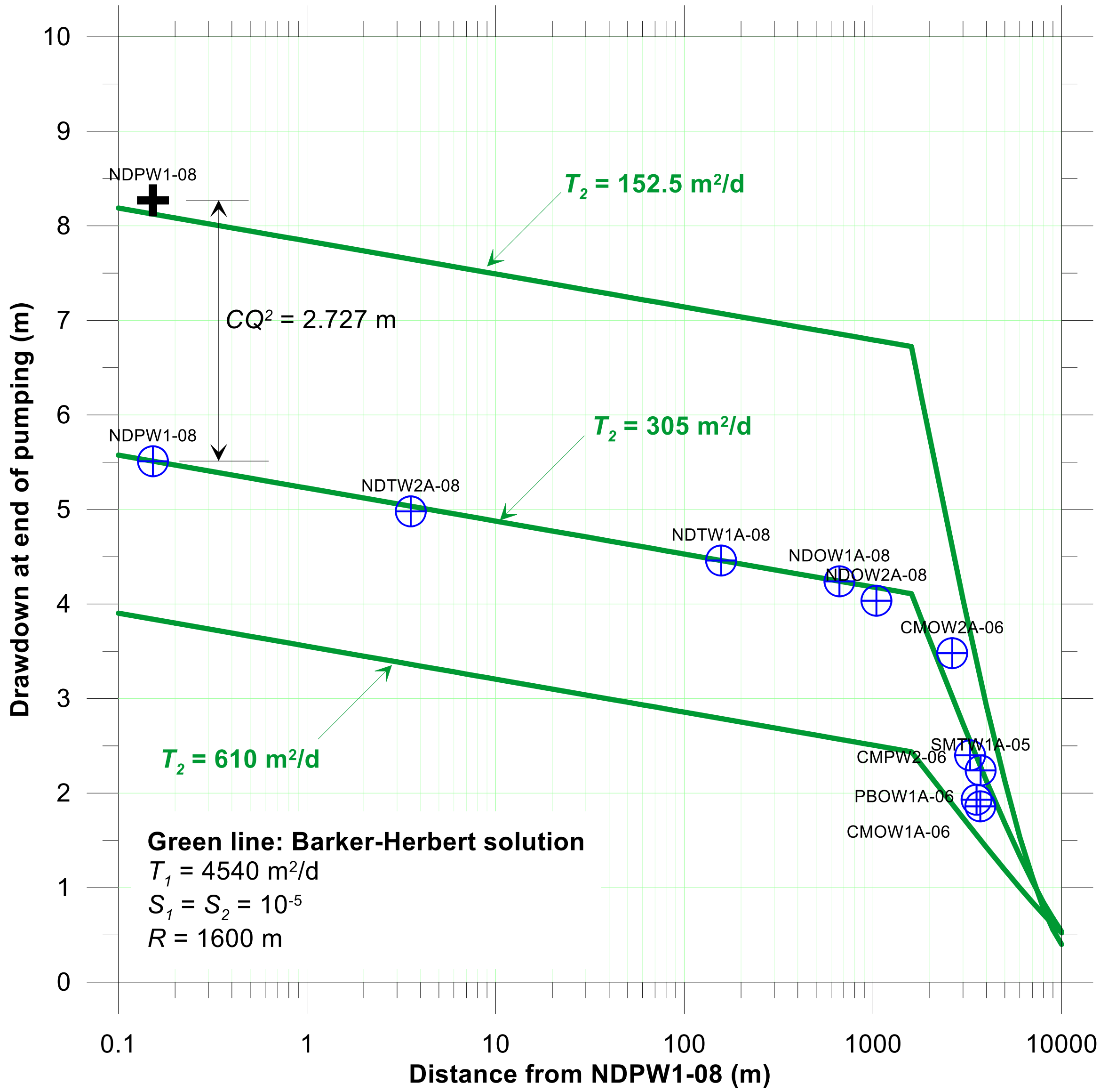

Figure 15. Barker-Herbert model, sensitivity analysis with respect to T_2

Pumping test case study: Elmira W3

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Last update: January 26, 2026

1. Introduction

The former Uniroyal chemical manufacturing facility in Elmira, Ontario is associated with one of the most prominent cases of groundwater contamination in Canada (Whiffin and Rush, 1989; Belanger and others, 1990; Belanger and others, 1992). In late 1989, low concentrations of a toxic compound, N-Nitrosodimethylamine (NDMA) were detected in groundwater samples taken from two municipal wells in the town’s south well field. The well field is located in a sand and gravel unit referred to as the Municipal Aquifer. The concentrations exceeded action levels and the wells were shut down. A containment and treatment system for the Municipal Aquifer consisting of two extraction wells began operating in August 1998 (Polan and Quigley, 1998). Pumping tests were conducted at both wells to support the design of the remedial system and to constrain the groundwater model that was developed to predict the long-term performance of the remedial system. For this case study, we will review the interpretations of the 24-hour pumping test conducted at well W3.

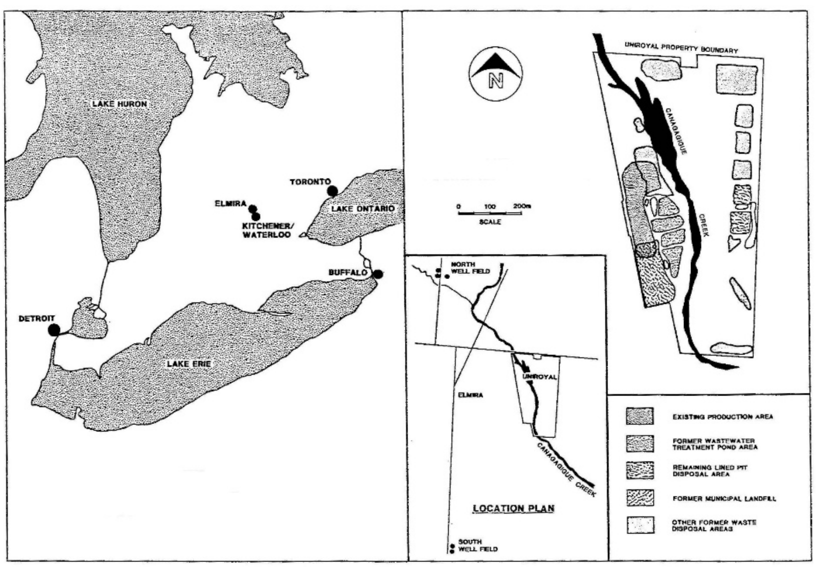

Figure 1. Site location (Whiffin and Rush, 1989)

2. Conceptual model

A map indicating the locations of the wells at the site is provided in Figure 2. The site map shows the locations of a north-south cross-section A-A’ shown in Figure 3, and an east-west cross-section C-C’ shown in Figure 4.

Groundwater flow in the Elmira area occurs primarily in the unconsolidated glacial sediments. The geology of the site consists of a complex sequence of glacial, glaciofluvial and glaciolacustrine deposits. The hydrostratigraphy is best described as a sequence of discontinuous aquifers and aquitards that have varying thicknesses and hydraulic conductivities (Morrison Beatty, 1985). The pumping well W3 is screened in the middle sand and gravel unit, the Municipal Aquifer (MA). The screened intervals for two of the observation wells, OW121 and OW118, are indicated in Figure 4. Not all of the observation wells are screened across the same stratigraphic unit as the pumping well.

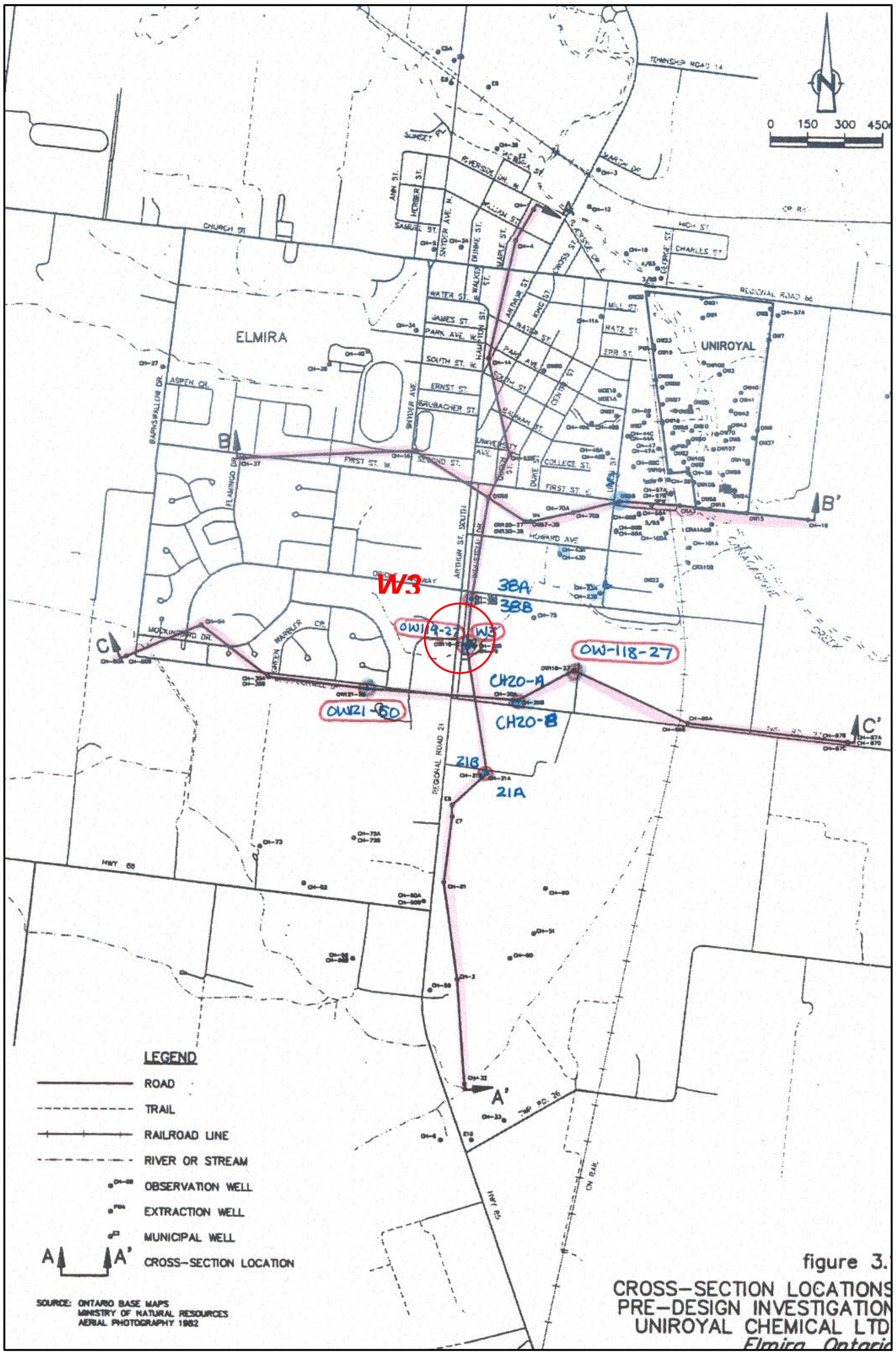

Figure 2. Locations of observation wells and cross-sections

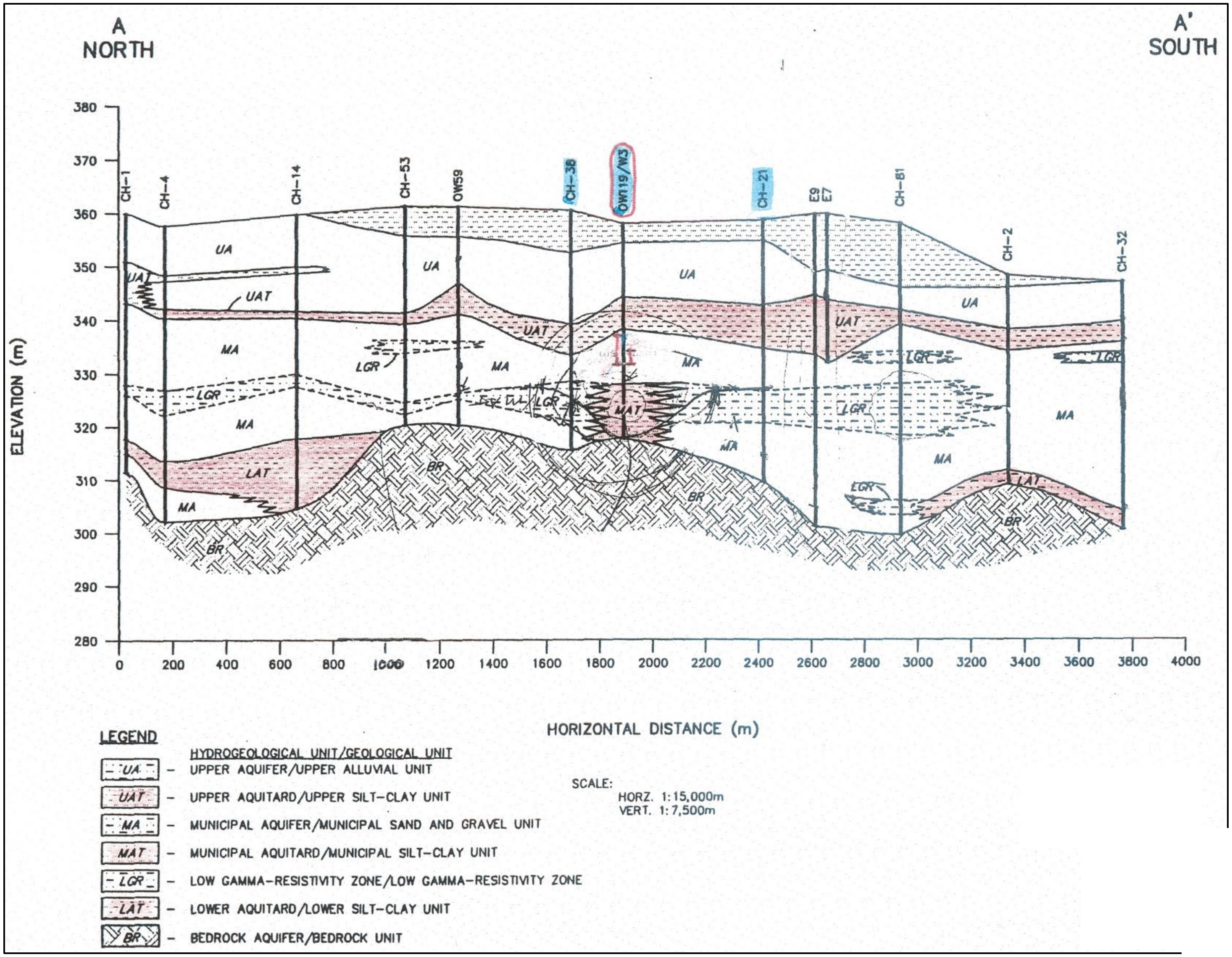

Figure 3. North-south cross-section A-A'

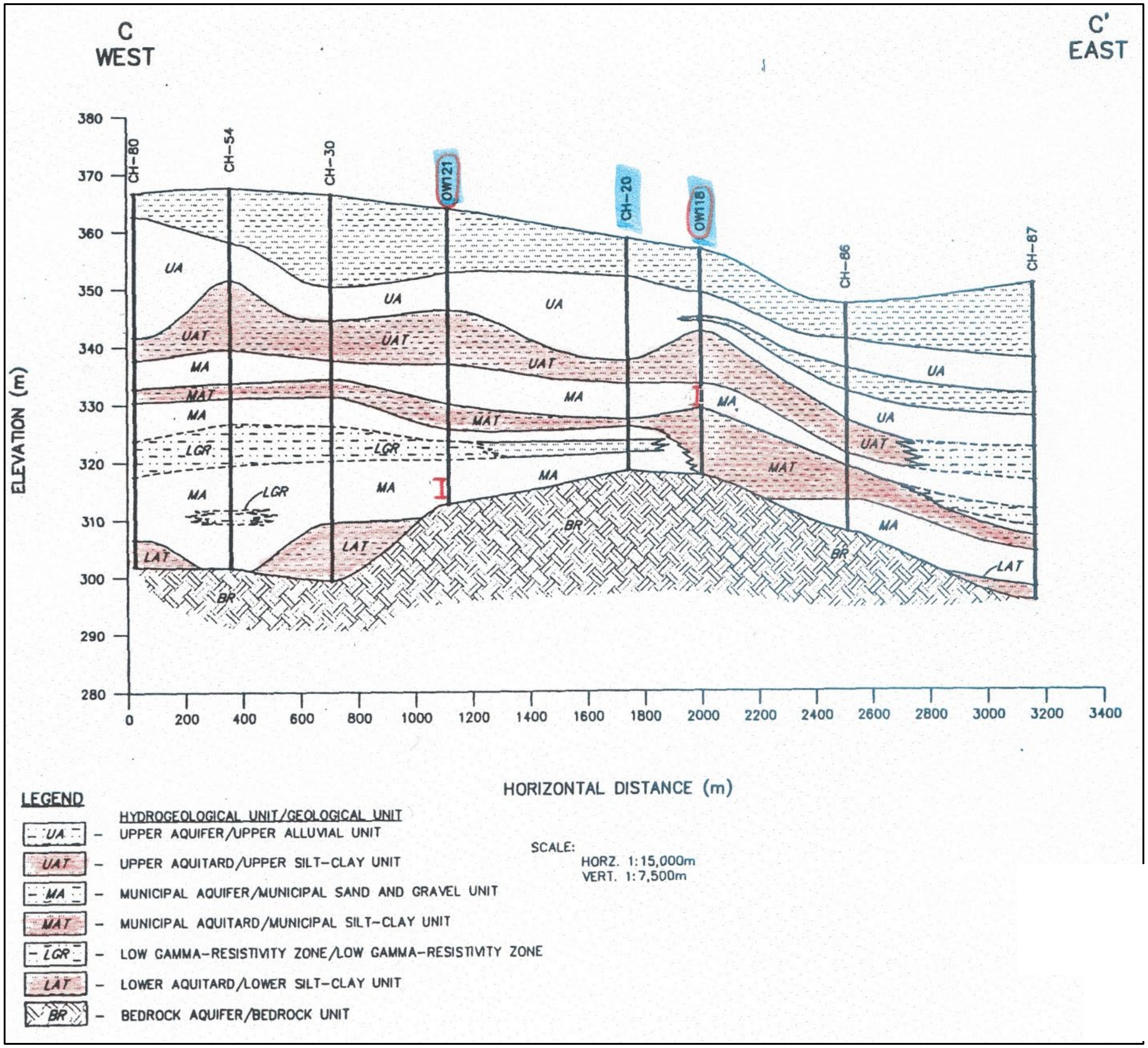

Figure 4. East-west cross-section C-C'

3. Original interpretation of the pumping test

Water levels were measured in the pumping well and 14 observation wells. The results of the pumping test were analyzed using a standard approach. In particular, separate analyses of the time-drawdown record were conducted for each of the observation wells, with the transmissivity estimated by matching the Theis (1935) solution to the observed drawdowns. The results from the analysis conducted for one of the wells is shown in Figure 5.

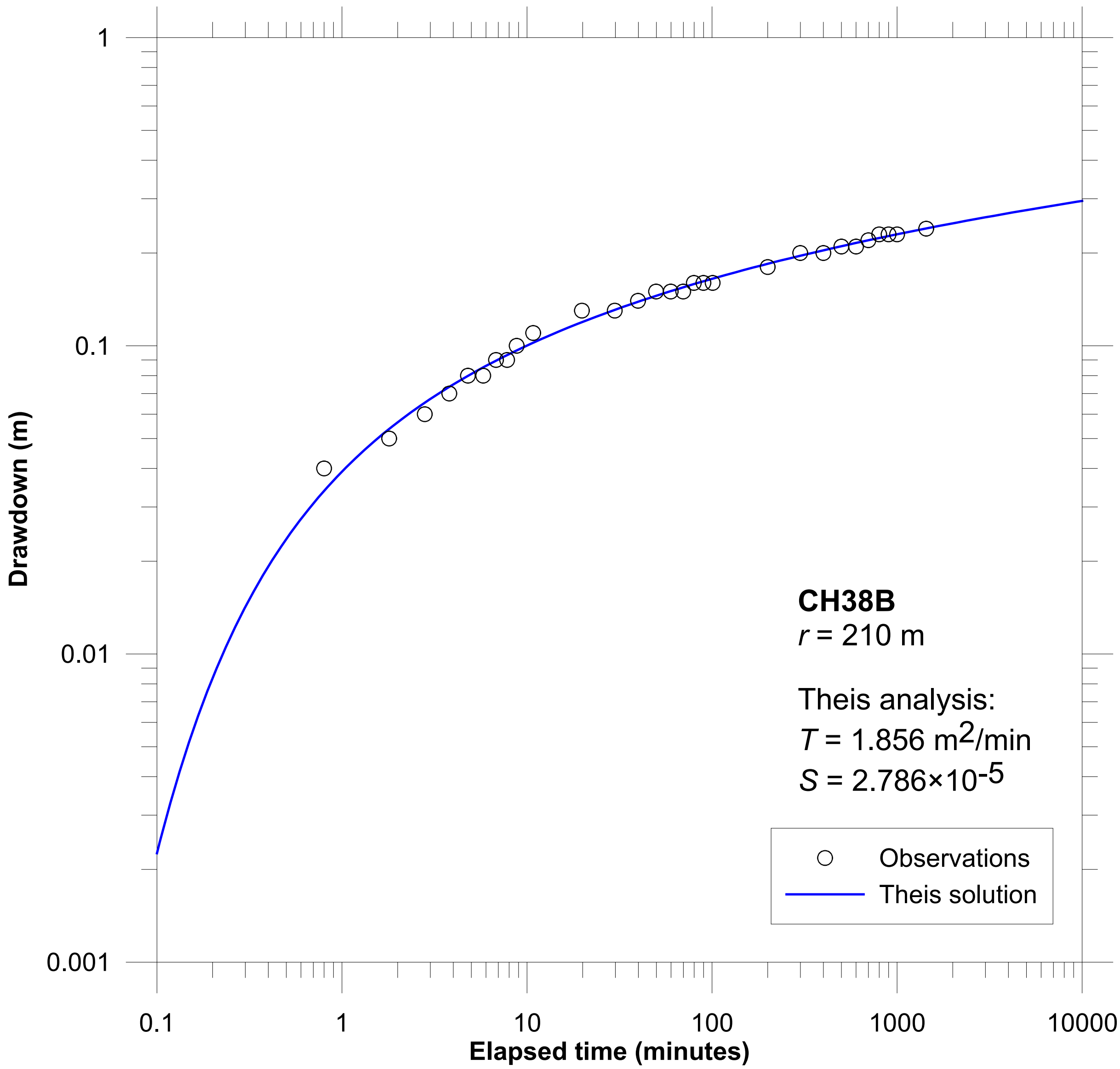

Figure 5. Analysis for one of the observation wells

The summary of the results of the pumping test interpretations presented in the consultant's report is reproduced below.

Location	Distance to Pumped Well (metres)	Drawdown at 23 Hours (metres)	Drawdown Results (m^2/min)	Drawdown Results (m^2/day)	Recovery Results (m^2/min)	Recovery Results (m^2/day)	Geometric Mean (m^2/min)
W3	0.127	1.26	0.5371	7.73E+02	1.0380	1.49E+03	7.47E-01
CH78B (1)	3	0.53	1.1360	1.64E+03	0.9562	1.38E+03	1.04E+00
OW119-27 (1)	35	0.71	0.9637	1.39E+03	0.9707	1.40E+03	9.67E-01
CH38A	210	0.36	0.9665	1.39E+03	0.9503	1.37E+03	9.58E-01
CH38B	210	0.24	1.7900	2.58E+03	1.7390	2.50E+03	1.76E+00
CH20B	240	0.07	3.8680	5.57E+03	6.4610	9.30E+03	5.00E+00
CH75A	290	0.37	0.6343	9.13E+02	0.5222	7.52E+02	5.76E-01
CH75B	290	0.43	0.7488	1.08E+03	0.6304	9.08E+02	6.87E-01
OW118-27	400	0.09	3.1000	4.46E+03	1.9540	2.81E+03	2.46E+00
OW121-50	450	0.17	2.0580	2.96E+03	1.9890	2.86E+03	2.02E+00
CH43A	490	0.25	0.9151	1.32E+03	0.8451	1.22E+03	8.79E-01
CH21A	545	0.06	4.5790	6.59E+03	2.3010	3.31E+03	3.25E+00
CH21B	545	0.05	11.1100	1.60E+04	7.6270	1.10E+04	9.21E+00
CH23B	575	0.30	1.1180	1.61E+03	0.9304	1.34E+03	1.02E+00
OW56-26	900	0.34	2.0380	2.93E+03	0.4996	7.19E+02	1.01E+00
					Geometric Average of all Results:		1.47E+00

Notes:

(1) The transmissivity value calculated from drawdown results has been corrected for partial penetration of the aquifer by the observation and pumping wells.

The pumping test was conducted at 150 Igpm (0.66 m^3/min) on July 20-21, 1994.

All transmissivities were obtained using the Theis method in AQTESOLV except the drawdown test for W3 which was analysed with the Cooper- Jacob method in AQTESOLV.

m^2/min metres squared per minute

m^2/day metres squared per day

4. Review of the original interpretation

The analyses that we have described thus far are "standard" in the sense that they follow a methodology that is typically adopted in practice. However, that does not make the results reproduced here reliable. In fact, we will argue that the analyses are an excellent demonstration of how not to report the interpretations of a pumping test, and how not to proceed with an analysis. We have four main objections:

- The reporting is inappropriate;
- The analysis approach is not conceptually sound;
- There are no indications of the relative reliability of transmissivity estimates; and
- Some of the interpretations are likely erroneous.

4.1 The reporting is inappropriate

We should retain as much precision in our intermediate calculations as possible. However, we should be very careful in how we report our results. In the reporting reproduced here, the individual transmissivity estimates are reported with 5 significant figures. In our opinion, transmissivity estimates should never be reported with more than 2 significant figures. We do not want to provide a misleading impression of the accuracy of our analyses, and we certainly don't want to advertise that we have little physical appreciation of the "exactness" of our interpretations. The transmissivity estimates are in no way exact, and with this inappropriate reporting we are left wondering whether the analyst understands the difference between precision and accuracy.

4.2 The analysis approach is not conceptually sound

The fundamental assumption underlying the Theis analysis is that the transmissivity is uniform. Therefore, to be consistent with the underlying conceptual model, our analyses of the individual time-drawdown records should have yielded the same transmissivity estimate. In this case, as many values of transmissivity are reported as there are observation wells.

Many hydrogeologists take the estimation of multiple values of transmissivity as "proof" that the aquifer is heterogeneous. The aquifer may indeed be heterogeneous, but the only thing that is really proved is that the results of the analysis contradict the fundamental assumption of the model adopted for the analyses. There is no guarantee that any of the multiple transmissivity estimates are representative. The reporting of multiple estimates of transmissivity demonstrates only that the analyst has either adopted an incorrect conceptual model or made inappropriate analyses.

In the reporting, the transmissivity for each observation well is reported as the geometric mean of the estimates derived from the drawdown and recovery analyses. There is no physical basis for calculating a geometric mean or any other average. For an analysis that is internally consistent, the transmissivities estimated for the drawdown and recovery portions of the test should be close. Obtaining different values is only demonstrates further that a fundamental aspect of the analysis is wrong.

The final reported transmissivity is the "Geometric average of all results". This is calculated as the geometric mean of the average transmissivity estimate for the individual observation wells. There is no physical basis for this averaging either. For an analysis that is internally consistent, the transmissivities estimated for each observation well should be the same, not following a lognormal distribution. Once again, the different values only demonstrate that a fundamental assumption of the analysis is violated.

4.3 Some of the interpretations are likely erroneous

The preceding critique might have alerted us to the possibility that some or all of the interpretations may be questionable. Let us take a closer look at some of the data. Our examination will reveal that although the calculations may have been correct, some of the interpretations are downright erroneous. We will examine the responses at two observations located approximately the same distance from the pumping well:

- CH38B, $r = 210$ m
- CH20B, $r = 240$ m

The drawdowns records are plotted in Figure 6. Without conducting any analyses, what can we say about the relative transmissivity inferred from the responses at both observation wells? Is the material around CH38B more or less transmissive than the material around CH20B? The answer to this question might appear to be obvious. The response at CH20B evolves more slowly and is attenuated compared to CH38B. Intuitively, we would expect that CH20B is located in a zone of less transmissive material.

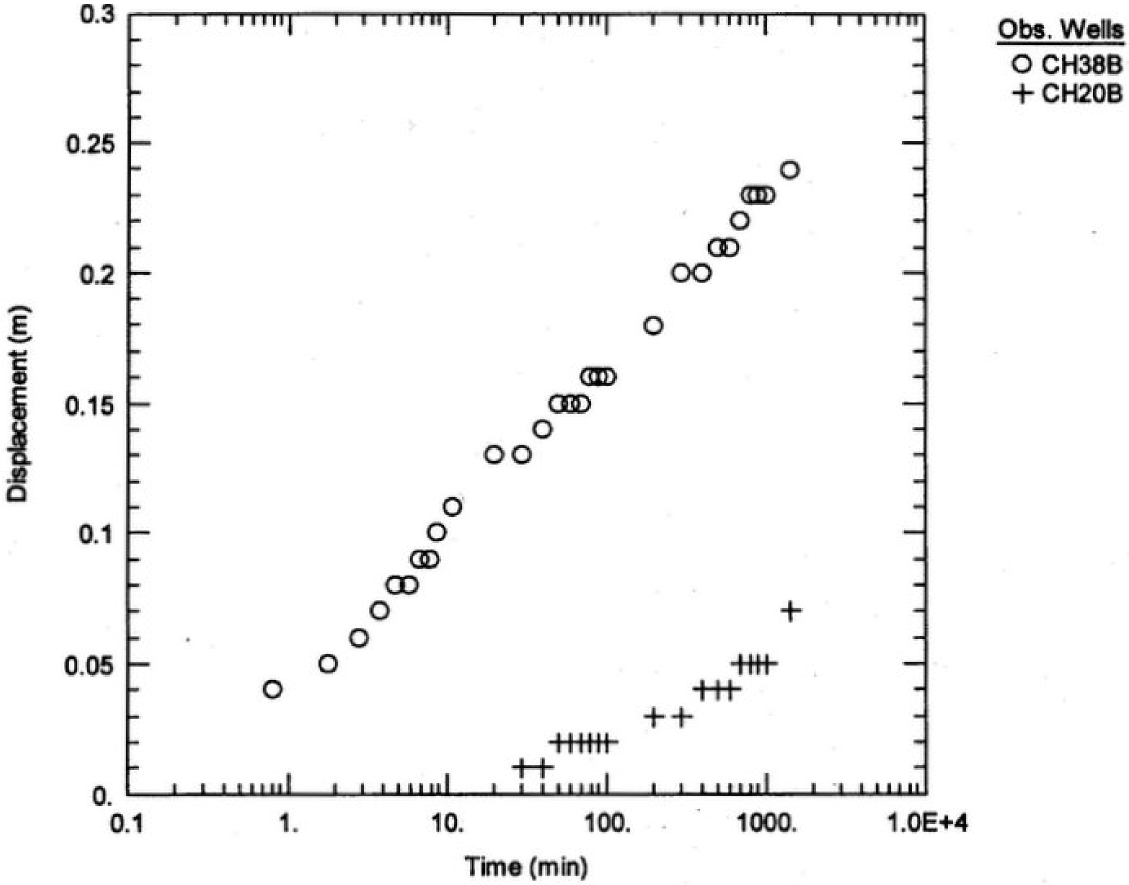

Figure 6. Results for wells 210 m and 240 m from the pumping well

To test our intuition, let us conduct conventional pumping test analyses with the Cooper-Jacob approximation of the Theis solution. Semilog plots of the drawdowns at CH38B and CH20B are shown in Figure 7.

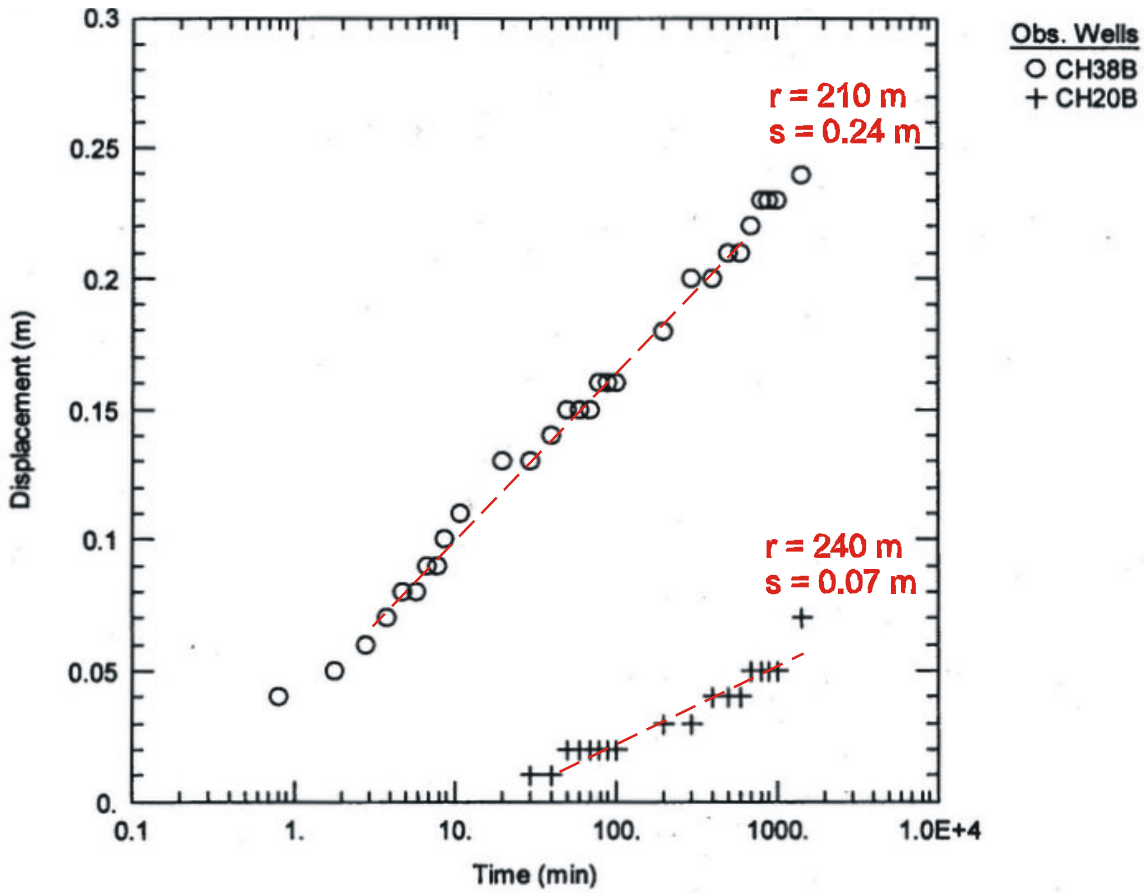

Figure 7. Cooper-Jacob analyses for wells 210 m and 240 m from the pumping well

<u>Cooper-Jacob straight-line analyses</u>

CH38B:

$$T = 2.303 \frac{Q}{4\pi} \frac{1}{\Delta s}$$

$$= 2.303 \frac{\left(0.66\ \text{m}^3/\text{min}\right)}{4\pi} \frac{1}{\left(0.061\ \text{m}\right)}$$

$$= 2.0\ \text{m}^2/\text{min}$$

CH20B:

$$
\begin{aligned}
T &= 2.303 \frac{Q}{4\pi} \frac{1}{\Delta s} \\
&= 2.303 \frac{\left(0.66 \text{ m}^3/\text{min}\right)}{4\pi} \frac{1}{\left(0.029 \text{ m}\right)} \\
&= 4.2 \text{ m}^2/\text{min}
\end{aligned}
$$

The results from the Cooper-Jacob straight-line analyses suggest that the material around CH20B is more transmissive than the material around CH38B. Are these results consistent with our expectations? If they aren't, which is flawed: our intuition or the interpretations?

4.4 There are no indications of the relative reliability of transmissivity estimates

A key reason we conduct a pumping test is to try to learn something about the structure of the subsurface. Our objective is not to obtain a table of transmissivity estimates. Rather, it is to understand the characteristics of the site. The averaging procedures used in the reporting assume that all of the estimates are equally reliable. It is not possible to tell from the summary table which transmissivity estimates may be most representative of the bulk properties of the formation. In our opinion, the analyses have no diagnostic value. In particular:

- The table of reported values does not shed any light on the subsurface structure;
- The table of reported values does not provide any insight into the representative large-scale transmissivity; and
- The table of reported values does not help to identify outliers.

5. Alternative interpretation approach

As a first step in our alternative interpretation, let us assemble all of the drawdown data on a semilog composite plot.

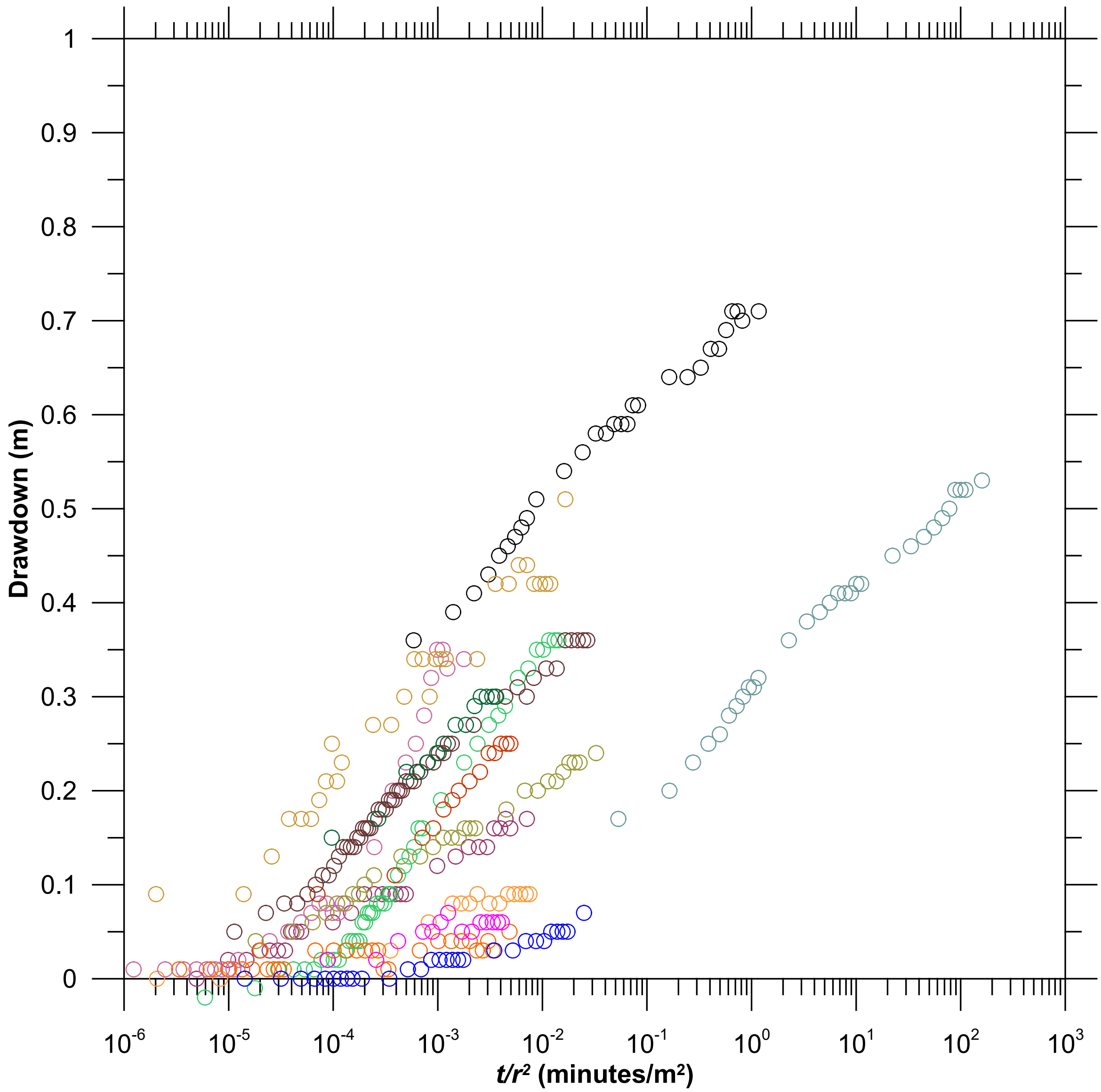

Figure 8. Composite plot of drawdowns observed during W3 pumping test

The complexity of the subsurface at Elmira is clearly evident when we superimpose all of the individual time-drawdown records on a composite plot in Figure 8. We can use the composite plot to identify those wells that have similar responses. The composite plot suggests that some wells show relatively little response. These wells are highlighted in Figure 9. It is not obvious why these wells appear to respond differently. It may be that the wells are screened in an aquifer that is different from the aquifer in which the pumping well is screened. It is also possible that the wells are screened in the same aquifer, but in zones that are somewhat less permeable. Rather than conceiving of these wells as outliers, it may be more appropriate to infer that the response at some of the wells evolves more slowly.

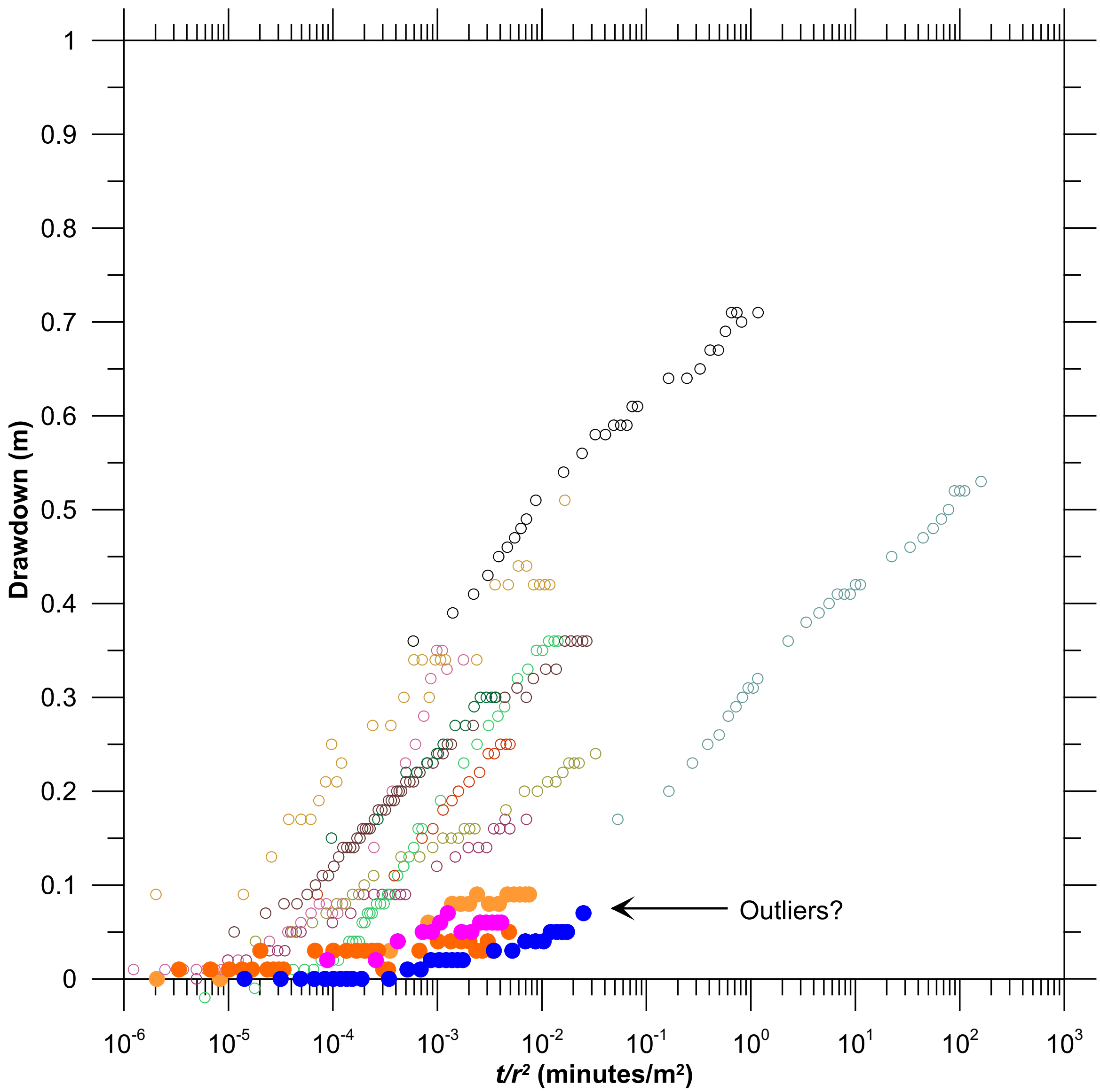

Figure 9. Composite plot of drawdowns observed during W3 pumping test

Estimation of transmissivity

The most reliable interpretations are generally based on a consideration of data collected after a relatively long time. The Cooper-Jacob analysis is ideal for identifying this portion of the response. The complexity of the responses to pumping shown in Figure 8 does not necessarily preclude the application of the Theis model. Butler (1990) and Meier and others (1998), among others, have demonstrated that reliable estimate of the large-scale transmissivity of a heterogeneous formation can be obtained with applications of the Cooper-Jacob straight-line analysis. However, the best approach for accomplishing our objectives for a more reliable analysis of the W3 pumping test is to conduct the analysis directly with the composite plot.

In Figure 10, we have plotted parallel straight lines through those responses that we believe provide insight into the bulk response of the aquifer. These are not lines of best-fit. Rather, they are lines that pass through a portion of the data and that have the same slopes. By identifying responses with similar slopes on the semi-log composite plot, we have armed ourselves with everything we need to estimate a representative "bulk" transmissivity Cooper-Jacob straight-line analysis. We recall from the Cooper-Jacob analysis that lines with the same slope yield the same estimate of transmissivity. Therefore, our analysis is explicitly consistent with the underlying assumption that the transmissivity is uniform.

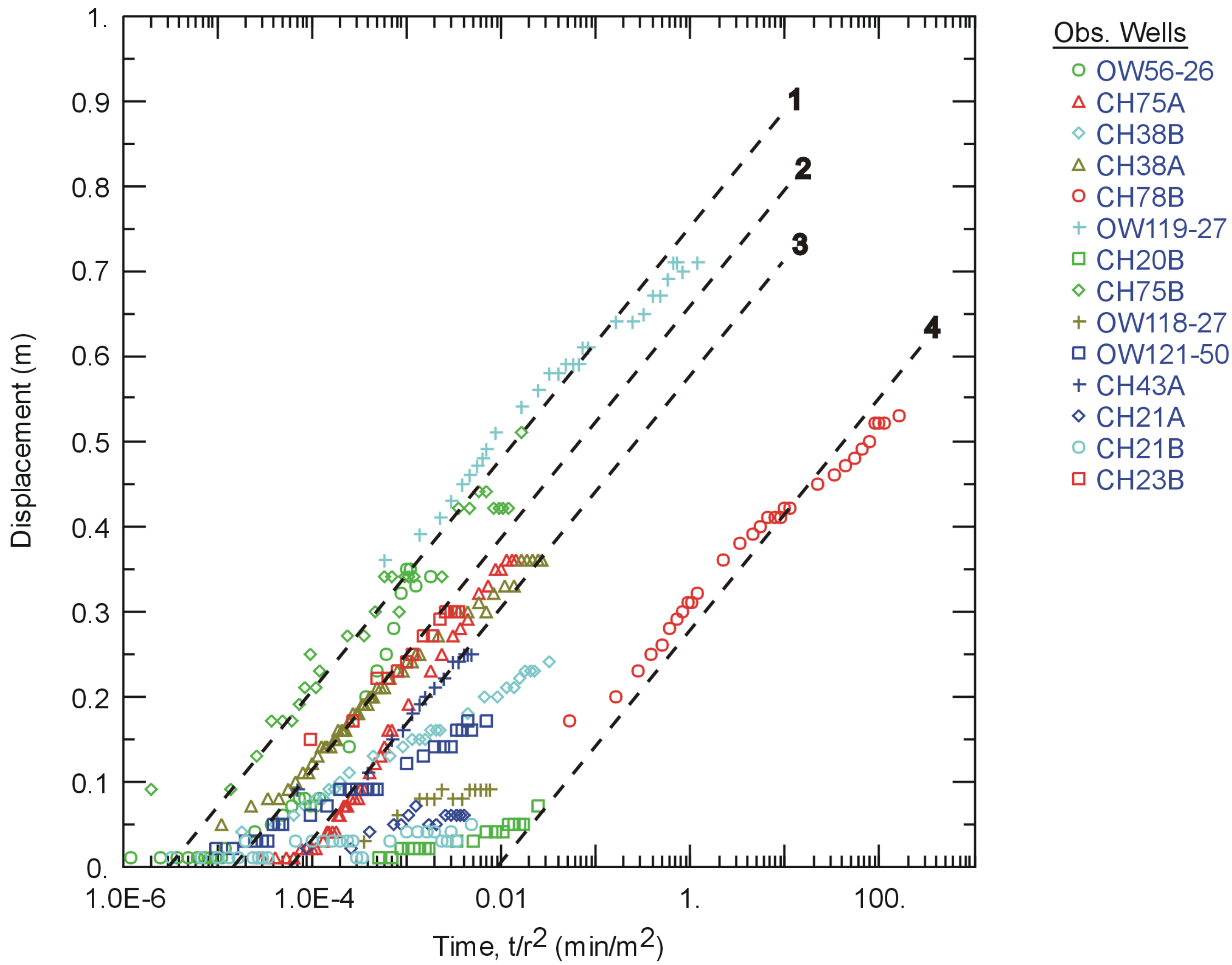

Figure 10. Cooper-Jacob straight-line analysis – identification of common responses

From the common slope a single transmissivity value is estimated:

$$
\begin{aligned}
T &= 2.303\frac{Q}{4\pi}\frac{1}{SLOPE} \\
&= 2.303\frac{\left(0.66\ \mathrm{m^3/min}\right)}{4\pi}\frac{1}{\left(0.27\ \mathrm{m/log\ cycle}\ t/r^2\right)} \\
&= 0.45\ \mathrm{m^2/min} \leftarrow
\end{aligned}
$$

6. Assessment of the alternative interpretation

We have adopted a deliberately simplified approach towards the interpretation of the results from the Elmira W3 pumping test. The approach we have adopted is certainly less involved the analysis of the individual time-drawdown records. In spite of the simplicity of the approach, our analysis has several important aspects that are missing from the original analysis:

- We have considered all of the data simultaneously;
- We have identified those drawdown records that appear to be representative of the bulk response of the aquifer, and those that appear to be outliers; and
- We have developed a single estimate of the bulk transmissivity for the aquifer.

<u>How does our estimate of the bulk-average transmissivity compare with the results of the individual Theis analyses?</u>

A bulk-average transmissivity of 0.45 m^2/min has been estimated from the Cooper-Jacob straight-line analysis with the composite plot. How does this estimate compare with the results of the individual Theis analyses? The results of the individual analyses are assembled in Figure 11. The red dashed line in the figure denotes the estimate from the composite plot. We see that the value of 0.45 m^2/min is reasonably consistent with the lower estimates inferred from the analyses of individual wells. More importantly, we see that the transmissivities estimated for CH20B, OW118-27, CH21A and CH21B are likely not representative of the bulk-average properties of the formation.

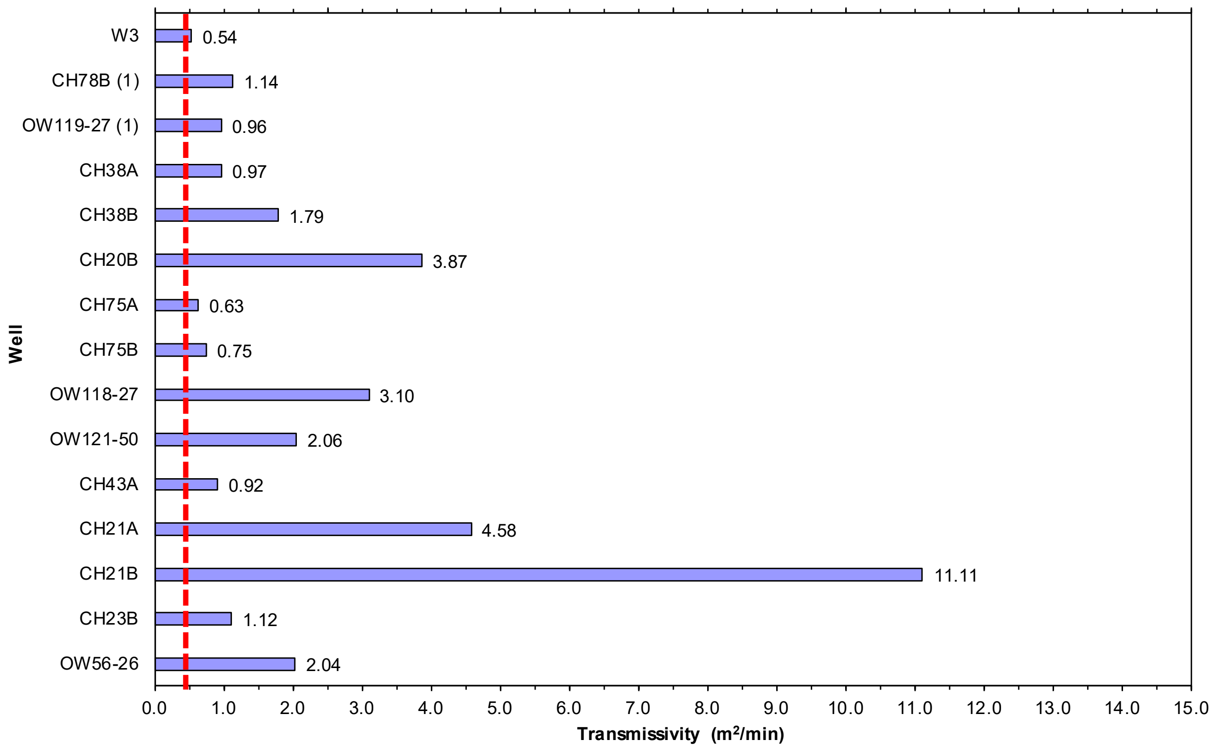

Figure 11. Illustration of individual transmissivity estimates

What else could we include in the interpretation?

One of the things we might do to improve our interpretation is to study the geologic logs for the site. In Figure 12, we present simplified logs for the pumping well and three observation wells. As just one example of the complexity, it is possible that the gravel in which well OW121-50 is screened may lie below the gravel in which W3 is screened.

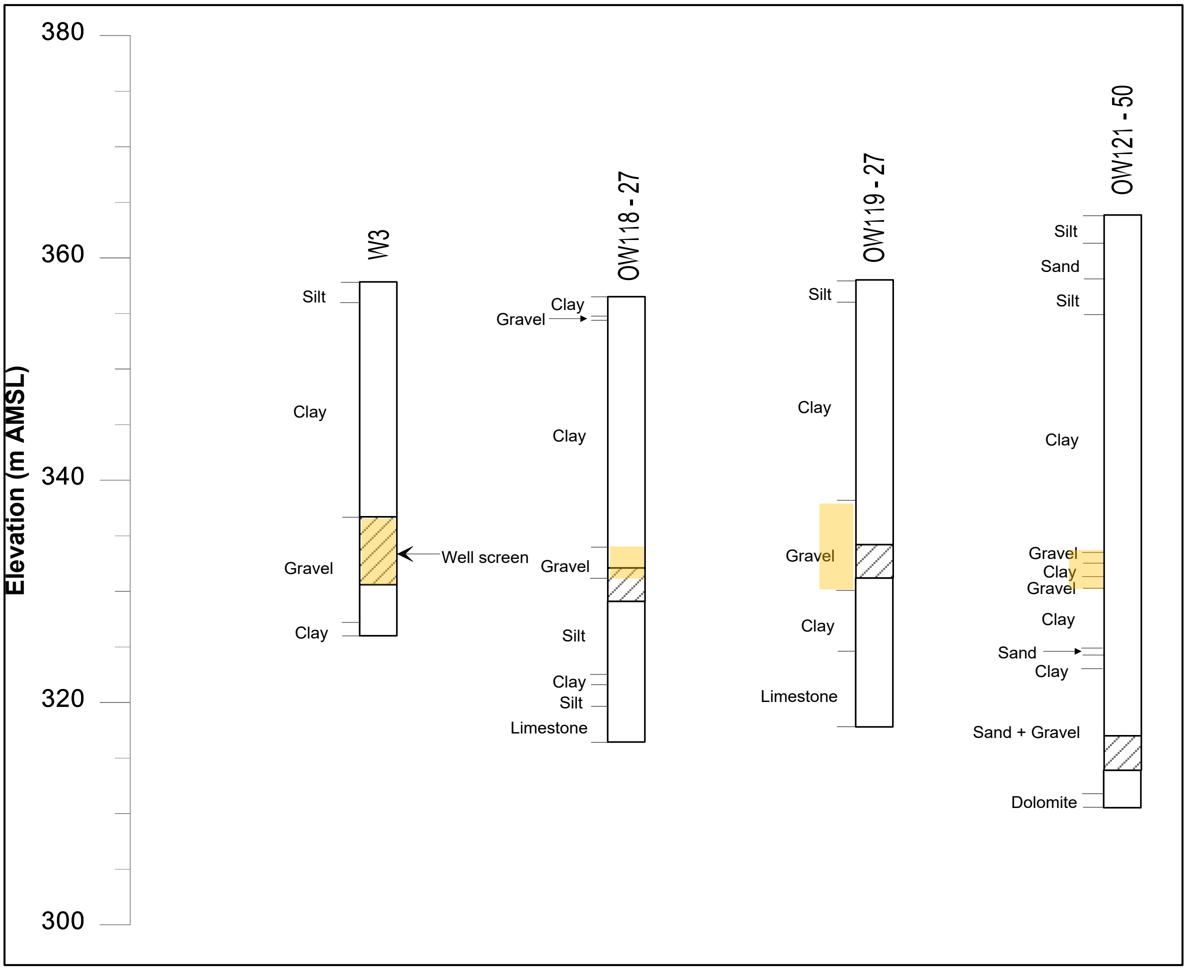

Figure 12. Simplified geologic logs and well completion details for four wells

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The Role of Pumping Tests in Site Characterization: Some Theoretical Considerations

by James J. Butler, Jr.[a]

Abstract

Pumping tests are the primary means of estimating the large-scale storage and transmissive properties of an aquifer for site-characterization investigations. Most analyses of pumping-induced drawdown are performed using either the Theis log-log curve-matching procedure or the approximate Cooper-Jacob semilog method. These two procedures provide dissimilar estimates in nonuniform aquifers due to their emphasis on properties in different portions of a unit. The log-log curve-matching approach heavily weights the properties of local material, while the semilog procedure emphasizes the properties of material within the front of the cone of depression. The different emphasis of the two procedures results in log-log parameters being more appropriate for estimating pumping-well drawdown, while semilog parameters are better for estimating well yield. The magnitude of the difference between parameters estimated by the two approaches is a function of the degree of aquifer nonuniformity and the distance between the observation and pumping wells. The further the observation well is from the pumping well, the smaller the difference between the parameters. The difference between parameters estimated by slug tests and those estimated by pumping tests, on the other hand, will increase with this distance. Due to their emphasis on near-well materials, slug-test parameters may be of use in estimating pumping-well drawdown when employed in a patchy aquifer model. In general, predictions of aquifer behavior can be improved by more careful application of the conventional techniques used in pumping-test analyses.

Introduction

The pumping test has traditionally been one of the primary field methods used by hydrogeologists to improve their understanding of conditions in the subsurface. This technique can provide several types of information to the hydrogeologist, such as conditions within, and in the immediate vicinity of, the pumping well (e.g., step-drawdown tests; Lennox, 1966), the large-scale flow behavior in the system (e.g., the nature of the vertical and lateral boundaries), and estimates of the transmissive and storage properties of the aquifer (Walton, 1970; Kruseman and DeRidder, 1970). This article addresses the role of pumping tests in providing estimates of subsurface flow properties for site-characterization investigations. Such parameter estimates can aid in assessing the effectiveness of various proposed remediation schemes as well as in predicting the gross movement of a contaminant plume. The focus of this article is on the hydraulic behavior of confined aquifers, although the conclusions are certainly not limited to such systems.

There are a number of techniques that can be used to analyze pumping-induced drawdown in confined aquifers. These include the log-log curve-matching approach first proposed by Theis in the late 1930s (Jacob, 1940), the semilog straight-line approach of Cooper and Jacob (1946), and the closely associated recovery analysis of Theis (1935), the pressure-derivative techniques primarily employed in petroleum engineering (Tiab and Kumar, 1980; Bourdet *et al.*, 1983), and various numerical models. Pressure-derivative techniques, which involve use of the temporal derivative of drawdown as the plotted quantity, have not been employed frequently in hydrogeological applications due to their sensitivity to noisy data. The use of numerical models is unjustified for many pumping-test applications because limited drawdown data make it difficult to move beyond the detail and accuracy of techniques based on analytical solutions. For most applications, the log-log curve-matching approach (henceforth designated the log-log approach) and the semilog straight-line method (henceforth designated the semilog approach) are the preferred methods for analysis of pumping-test drawdown. This discussion

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examines how a careful application of these conventional techniques may provide further insight into the nature of the transmissive and storage properties of an aquifer.

The basic message of this paper is that the log-log and semilog methods may provide different estimates of flow properties in nonuniform aquifers. The term "nonuniform" is used here to designate units with spatially varying flow properties, such as those found in natural systems. Although parameters determined by these two methods in hypothetically uniform aquifers will be equal, this equality will not generally hold in nonuniform systems. The purpose of this paper is to discuss why this inequality exists, what it means for parameter estimation, when knowledge of the potential for this inequality will be important for practicing hydrogeologists, and how this knowledge can be used to improve estimates of pumping-well yield and drawdown.

Relatively few workers have examined the question of the viability of conventional techniques for pumping-test analysis in nonuniform aquifers (e.g., Warren and Price, 1961; Vandenberg, 1977; and Barker and Herbert, 1982). Butler (1986) used Monte Carlo simulation to demonstrate that the log-log and semilog methods provide dissimilar estimates in aquifers whose property variations can be represented by stationary stochastic processes. In order to explain the differences between these estimates, he interpreted pumping-induced drawdown as consisting of two components: one dependent on near-well material, and one independent of such material. A large body of work in the petroleum engineering literature provides support for this interpretation (see Streltsova, 1988, and references therein). The log-log and semilog methods were hypothesized by Butler (1986) to be emphasizing these two components of drawdown in a dissimilar manner. This hypothesis is considered in greater detail here.

Drawdown at a Pumping Well

The first step in explaining why the log-log and semilog analyses produce different estimates in nonuniform aquifers is to consider the nature of drawdown at the pumping well in some detail. Although several recent publications have explored this issue in considerable depth (see above references), discussions of this topic have occurred often in the literature over the last 50 years. Theis (1940) was one of the first to speculate on the nature of pumping-induced drawdown, noting that at large durations of pumpage, the portions of the aquifer in the vicinity of the pumping well provide an insignificant contribution to well discharge. He stated that, at those times, the portions of the aquifer in the vicinity of the pumping well serve merely as conduits to transport water from more distant regions. He defined the portions of the aquifer that provide the majority of the water as those in which drawdown is not yet a linear function of the logarithm of the radial distance from the pumping well. In other words, the duration of pumpage has not been sufficient to allow the Cooper-Jacob approximation to be valid at those radial distances. This theory of Theis developed into the steady-shape, unsteady-state concept described by Heath and Trainer (1968) among others. Recently, Neuman (1987) has employed similar concepts to point out the difficulties in applying a water-balance approach to estimate the specific yield of an unconfined aquifer.

The portion of the aquifer controlling changes in drawdown at any given time during a pumping test is essentially a concentric ring of material that continually increases in width as it moves away from a pumping well in an infinite aquifer. This ring is designated here as the front of the cone of drawdown (depression). If the front of the cone of drawdown is defined as that portion of the aquifer that is contributing 95% of the flow to the pumping well, the inner and outer radii of this ring can be defined as follows (Streltsova, 1988):

$$r_{inner} = \sqrt{(.1)Tt/S} \qquad (1)$$

$$r_{outer} = \sqrt{(14.8)Tt/S} \qquad (2)$$

where T = transmissivity, $[L^2/T]$; S = storativity, dimensionless; and t = duration of pumpage, [T]. Note that this discussion strictly holds true only for uniform or radially nonuniform [the diffusivity (T/S) of (1) and (2) is defined differently in this case] systems. Although the concentric ring will be distorted, the concept is a very good approximation for most nonuniform systems encountered in the field.

Figure 1 illustrates the primary points of the preceding discussion. Figure 1a displays conditions at a very early time of pumpage. Since the pump has only been on for a short period, the front of the cone of drawdown has not had time to move away from the well. Therefore, the flow

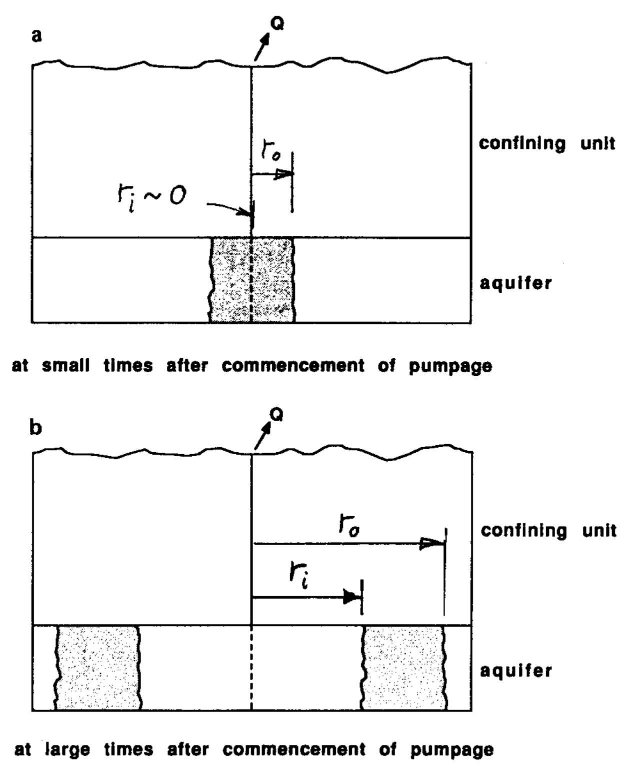
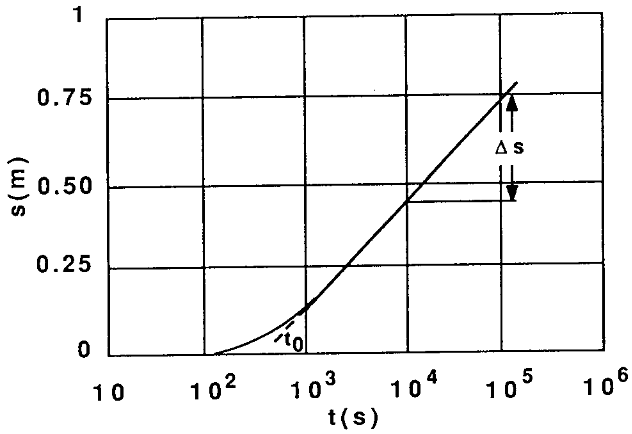


Fig. 1. Cross-sectional view of a confined aquifer at different times during pumping; stippled region indicates portions of aquifer controlling changes in drawdown at the specified time (not to scale).



$$T = \frac{2.3Q}{4\pi\Delta s} \qquad S = \frac{2.25Tt_0}{r^2}$$

Fig. 2. Procedure employed in the semilog method for calculation of aquifer parameters (after Freeze and Cherry, 1979).

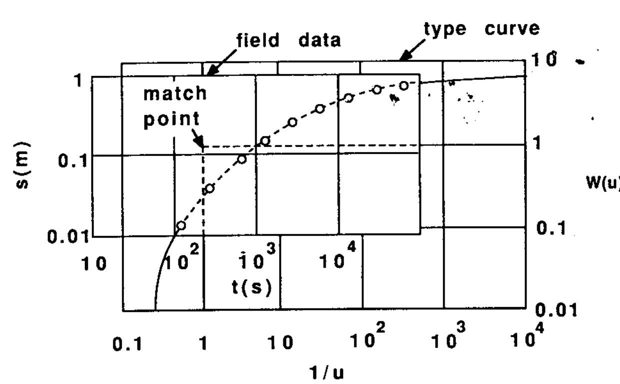
properties of the near-well portions of the aquifer will be an important control on changes in drawdown during the period following the commencement of pumpage. If the pumping well has been sited in an area of rather anomalous properties or if a well skin has developed during drilling or development, the drawdown at the pumping well will reflect that situation.

Figure 1b displays conditions in the same aquifer after the pump has been on for a considerable period of time. In this case, changes in drawdown at the pumping well are still being controlled by the flow properties of material within the front of the cone of depression. The cone front, however, has now moved some distance from the pumping well. The properties of the aquifer between the inner radius of the cone front and the pumping well have a negligible effect on changes in drawdown at the pumping well during this period.

The implications of the behavior illustrated in Figure 1 for pumping-test analysis are relatively straightforward. Figure 2 is a review of the semilog straight-line procedure. In this approach, the slope of the straight-line portion of the semilog plot, i.e., the change of drawdown during a certain interval of time, is the quantity used to calculate transmissivity. As discussed above, this change in drawdown is a function of the transmissivity of material in the front of the cone of depression. Therefore, the estimated transmissivity is independent of the material between the inner radius of the cone front and the pumping well. This is one of the major strengths of the semilog technique; after just a short duration of pumpage, the calculated transmissivity is independent of any material of anomalous properties in the vicinity of the pumping well, of a well skin created during drilling or development, and of any well losses. The storage parameter, however, has been shown by Butler (1988) to be dependent on the variations in transmissivity between the pumping well and the front of the cone of depression. The dependence of the storage parameter on the transmissivity of the material between the pumping well and the front of the cone of depression arises from the use of the x-intercept of the semilog plot (t_0 of Figure 2) in the analysis.

Figure 3 is a review of the log-log curve-matching procedure. The important point to note about this approach is that total drawdown, and not its change during a certain interval, is the quantity of interest. This total drawdown can be influenced heavily by a number of factors including anomalous material in the vicinity of the pumping well, a well skin due to drilling or development, and head losses in the pumping well. When employing drawdown from the pumping well in an analysis, it must be recognized that the measurements have been affected by these phenomena. Note also that the graphical curve-matching nature of the approach often results in the interval of greatest curvature, i.e., the interval controlled by near-well phenomena, being heavily emphasized. Due to the dependence of the methodology on conditions in the vicinity of the pumping well, the spatial variation of transmissivities calculated from log-log analyses (log-log transmissivities) using drawdown at different pumping wells in an aquifer should be considerably larger than that of semilog transmissivities. In terms of temporal variations, Butler (1986) has demonstrated that log-log transmissivities will change slowly with duration of pumpage due to the heavy weighting of near-well material, whereas the temporal variation of semilog transmissivities may be large if the front of the cone of depression moves into material of differing properties.

A simple example, based on earlier work by Barker and Herbert (1982), can help illustrate these concepts by examining the impact that a small disk of material of anomalous properties, centered on the pumping well, can have on pumping-well drawdown. Figure 4a displays a configuration consisting of a disk, 2 meters in radius, embedded in material of differing, although uniform, properties. A semianalytical solution, which has been developed by a number of workers (see Butler, 1988), is employed here to simulate the flow to a pumping well located at the center of the disk. For this example, the transmissivity of the disk is one order of magnitude lower than that of the surrounding material. Figure 4b shows a



$$T = \frac{QW(u)}{4\pi s} \qquad S = \frac{4uTt}{r^2}$$

Fig. 3. Procedure employed in the log-log method for calculation of aquifer parameters (after Freeze and Cherry, 1979).

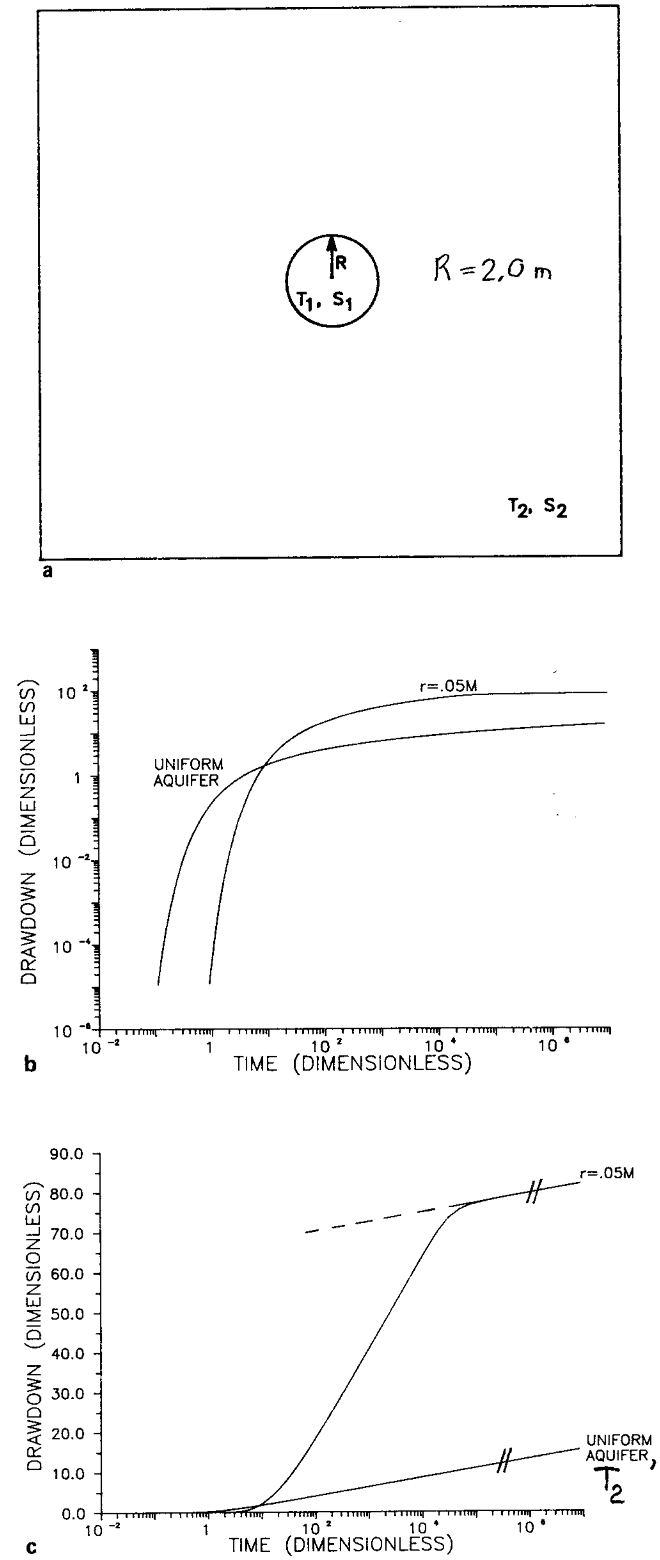


Fig. 4. a — Configuration consisting of a disk of material 1 (R = 2 m) embedded in material 2 ($T_1/T_2 = .1$, $S_1/S_2 = 1.0$); b — dimensionless log-log drawdown ($4\pi T_2 s/Q$) versus time ($4T_2 t/r^2 S_2$) plots for the embedded-disk and uniform-aquifer cases (r = radial distance from the center of the disk to the observation point, in this case can be considered the radius of a well centered at the origin); and c — dimensionless semilog drawdown versus time plots for the embedded-disk and uniform-aquifer cases.

simulated log-log plot of the drawdown that would be observed at the pumping well in this situation and the corresponding uniform-aquifer case. Figure 4c displays the semilog drawdown versus time relationships for this example. The parallel slopes of the two semilog plots at large times of pumpage demonstrate the independence of the semilog analysis from near-well material. Note that, although well-bore storage effects have not been incorporated into the simulations of Figure 4, the conclusions of relevance for this discussion are not dependent on such phenomena.

This example clearly indicates that, if the contribution of anomalous material is not recognized, a conventional log-log analysis using drawdown data from the pumping well may produce estimates that are not representative of average conditions in the aquifer. Traditionally, hydrogeologists have infrequently employed pumping-well drawdown in pumping-test analyses due to the recognition that the drawdown data have been affected by the phenomena mentioned above. These data, however, need not be ignored since a semilog analysis applied at moderate to large time is independent of the above phenomena. Petroleum engineers, not having the luxury of working with many observation wells, have long recognized the value of semilog plots for an analysis of production-well data [e.g., Earlougher (1977), Streltsova (1988), and references therein]. In fact, the concept of the infinitesimal well skin commonly employed in petroleum engineering developed in part from the observation of differences between log-log and semilog parameters (see Ramey, 1982, and references therein). Note that drawdown in the pumping well may be difficult to measure while the pump is in operation, so measurement during the post-pumpage recovery phase may be more appropriate. The above discussion concerning the semilog drawdown analysis also applies to a semilog recovery analysis (Theis, 1935).

Although infrequently used in pumping-test analyses, pumping-well drawdown is commonly employed to calculate the specific capacity of a well. Specific-capacity data are then often used to estimate aquifer transmissivity (e.g., Walton, 1970). Clearly, specific-capacity calculations will be impacted by the phenomena discussed here. Bradbury and Rothschild (1985) suggest an approach for correcting specific-capacity data for well losses. Additional attention, however, needs to be given to the impact of near-well materials on specific-capacity data.

Drawdown at an Observation Well

From the above discussion, it should be clear that the log-log and semilog analyses provide different parameter estimates in nonuniform aquifers. However, when pumping-test records from field tests are examined, a close correspondence between parameters calculated using these two approaches is often observed. This close correspondence seen in field data, which appears to contradict the above theoretical discussion, is primarily a result of the use of observation wells in most pumping tests. As is shown in the following paragraphs, the radial location of the observation well plays a very important role in the inequality between

$$t_D = \frac{4T_2 t}{r^2 S_2} = \frac{1}{u}(T_2, S_2) \; ; \quad s_D = \frac{4\pi T_2}{Q} s$$

the two sets of parameters. The primary reason for this is that the volume of the aquifer that is controlling the initial drawdown at an observation well increases significantly with distance from the pumping well. Assuming purely radial flow, one can calculate the volume of the aquifer that controls drawdown at different times during a pumping test using (1) and (2). This calculation indicates that the initial drawdown at an observation well at a considerable distance from the pumping well is controlled by such a large volume of the aquifer that the material in the vicinity of the observation well should have a very small impact on drawdown at that well.

In order to explore the influence of nearby material on observation-well drawdown in more detail, an analytical model can be employed. This model considers the case of pumping-induced drawdown in an observation well located in or near a disk of anomalous material, embedded in a uniform aquifer. The approach used here employs a recently developed analytical solution (Butler and Liu, 1989), which is based on earlier work by Jaeger (1944). This solution allows pumping-induced drawdown at any point within or in the vicinity of a disk of anomalous properties to be readily evaluated.

Figure 5 displays the configuration that is examined here. An observation well (O) is located at the center of a disk of radius R and properties T_{disk} and S_{disk}. The disk is assumed to be embedded in a uniform aquifer of properties T and S at a distance r from the pumping well (P). A constant discharge rate of Q is maintained at the pumping well throughout the period of analysis. Table 1 lists the four scenarios considered here. The first three scenarios describe conditions in which there is an order of magnitude difference between the transmissivity of the disk and that of the aquifer for disks of different radii. The fourth scenario describes a more extreme case when the transmissivity difference between the disk and the aquifer is two orders of magnitude. Figure 6 displays the results of the analysis in the form of log-log plots. Figure 6a depicts the results for the first scenario, indicating that for an observation well further than 5 meters from the pumping well, the effect of the disk is negligible. Figure 6b illustrates that drawdown at an observation well more than 10 meters from the pumping well is minimally impacted by a disk of 2 meters in radius.

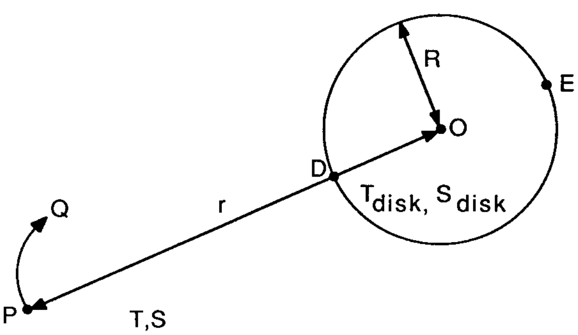


Fig. 5. Configuration employed in the analysis of the effect of near-well material on observation-well drawdown, notation explained in the text.

Table 1. The Four Scenarios Employed in the Analysis of the Effect of Near-Well Material on Observation-Well Drawdown

	R(m)	T_{disk}/T	S_{disk}/S
Scenario 1	0.5	0.1	1.0
Scenario 2	2.0	0.1	1.0
Scenario 3	5.0	0.1	1.0
Scenario 4	0.5	0.01	1.0

Figure 6c illustrates that a distance of over 25 meters is required before the effect of a disk of 5 meters in radius is negligible. Figure 6d displays the results from the more extreme case indicating that a very small area of anomalous properties can certainly affect observation-well drawdown several tens of meters from the pumping well. Figure 7 displays the results of the second scenario in a semilog plot format. This figure demonstrates the insensitivity of a semilog analysis to material in the vicinity of the observation well. Note that the plots of Figures 6 and 7 pertain to an observation well located at the center of a circular disk in order to consider the case of a disturbed zone created during drilling or development. An observation well located elsewhere in the disk or in its immediate vicinity would exhibit a slightly different drawdown response. Figure 8 displays the results of an analysis of drawdown at observation wells located at points D and E of Figure 5, using the third scenario of Table 1. As in Figure 6, the plots converge on the uniform-aquifer case with increases in distance from the pumping well and duration of pumpage. At large times, semilog drawdown plots will again be parallel to the uniform-aquifer curve. Although these statements are made with respect to a circular disk, they should pertain to a zone of any shape that is completely enclosed by material of differing properties. Note also, as with Figure 4, the conclusions of relevance for this discussion are not dependent on well-bore storage.

The general conclusion resulting from this analysis is that the further an observation well is from the pumping well, the less the drawdown is impacted by the properties of material in the immediate vicinity of the observation well. The specific distance from the pumping well at which a zone of anomalous properties will have a negligible impact on observation-well drawdown depends on the properties and extent of that zone. This conclusion implies that, in many cases, a well skin at an observation well will minimally impact drawdown. In those situations when near-well materials are considered to have an undue impact on observation-well drawdown, the semilog approach using moderate- to large-time data can be employed to remove the influence of those materials from transmissivity estimates. The semilog storativity, however, will still be dependent on the transmissivity between the observation well and the front of the cone of depression. This dependence will lessen as the observation well increases in distance from the pumping well. In general, aquifer parameters determined from drawdown at observation wells located at

 cf. Butler and Liu (1989)

The observation well is located a distance r from the pumping well.

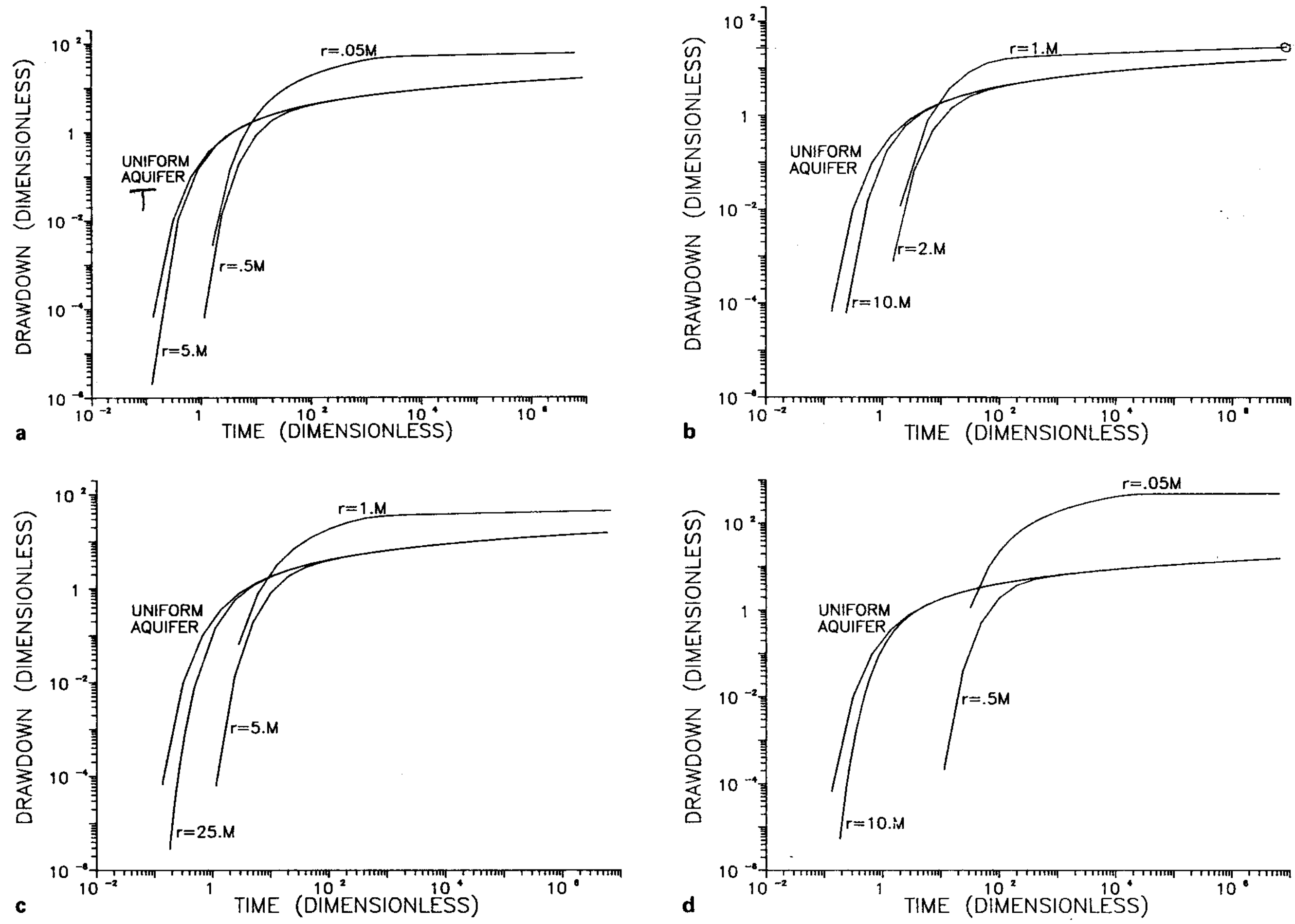


Fig. 6. Dimensionless log-log drawdown ($4\pi Ts/Q$) versus time ($4Tt/r^2S$) plots for the analysis of the effect of near-well material on observation-well drawdown: a — plots for the case of a disk of 0.5 meters in radius (T_{disk}/T = .1, S_{disk}/S = 1.0); b — plots for the case of a disk of 2.0 meters in radius (properties as in a); c — plots for the case of a disk of 5.0 meters in radius (properties as in a); and d — plots for the case of a disk of 0.5 meters in radius (T_{disk}/T = .01, S_{disk}/S = 1.).

a considerable distance from the pumping well should only be very weakly dependent on analytical methodology.

Except in the case of damage due to drilling or development or of an abrupt geologic boundary, the spatial

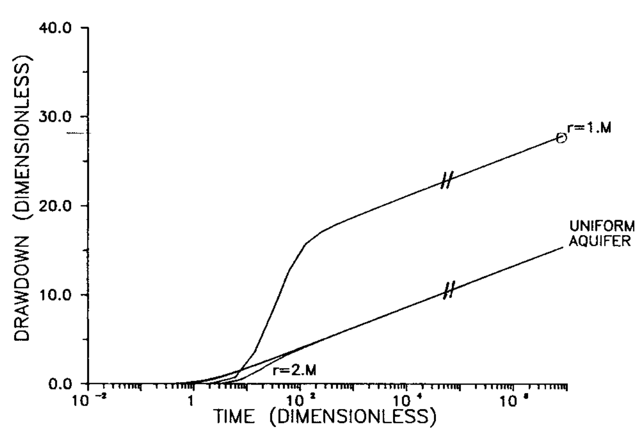


Fig. 7. Dimensionless semilog drawdown versus time plots corresponding to Figure 6b.

Curve for r=10 m plots on top of curve for uniform aquifer

variation in flow properties observed in most natural systems is more gradual than that employed in the above analysis (e.g., Smith, 1981; Sudicky, 1986). Butler (1986) used Monte Carlo simulation to assess behavior in units whose flow properties varied in a manner similar to that which might be expected in many natural systems (i.e., the spatial variations could be represented by a stationary stochastic process). He demonstrated that differences between log-log and semilog transmissivities as large as 30% would not be unusual in aquifers of a moderate degree of variability, when observation wells near the pumping well are used. In more variable systems, especially those characterized by a trend in flow properties, the difference could be considerably larger. As was demonstrated here, the further the observation well is from the pumping well, the smaller the difference between the two sets of parameters.

An additional point pertaining to the analysis of Figures 5-8 concerns the difference between transmissivities calculated from slug tests and those calculated from pumping tests. Given the configuration of Figure 5, an analysis of slug-test responses at the observation well would undoubtedly yield a transmissivity similar to that of the disk. Therefore, the difference between slug- and pumping-test parameters should increase the further the observation well is from the pumping well. Similarly, the distribution of slug-test transmissivities measured over a site should be much broader than the distribution of pumping-test transmissivities, which will depend on observation-well location and analytical methodology. Close agreement between slug-test and large-time semilog parameters at several observation wells should thus be considered strong support for representing that aquifer as a uniform unit at the slug-test or larger scale.

Drawdown Versus Yield

The importance of the difference between the two sets of parameters produced by the log-log and semilog approaches depends on the type of information desired from a pumping test. For the purposes of this discussion, the term "well yield" will be used to denote a reasonable maximum pumpage rate that can be sustained at a well in the absence of boundary effects. When an estimate of well yield is the focus of an investigation, the semilog analysis, with its emphasis on the properties at the front of the cone of depression, is clearly the preferred approach. A log-log analysis, using drawdown from an observation well at a considerable distance from the pumping well, can also supply a reasonable estimate of well yield in many systems. This is an important point as often the only wells at which drawdown can be measured are at such a distance from the pumping well that a semilog analysis is not applicable for a pumping test of limited duration. If an estimate of future drawdown at the pumping well is of interest, however, a transmissivity determined from a semilog analysis using moderate- to large-time data may prove of limited use. In that case, a transmissivity from a log-log analysis, using drawdown at or near the pumping well, is more appropriate. If estimates of both pumping-well drawdown and yield are desired, an observation point at or near the pumping well is recommended. Drawdown at such a well can then be analyzed using both the log-log and semilog approaches to provide parameters for estimating pumping-well drawdown and yield, respectively. Note that if drawdown at the pumping well is employed in the analysis, some effort should be made to correct for well losses so that log-log parameters independent of flow rate can be obtained. Also, log-log matching procedures that allow skin effects to be characterized (e.g., Gringarten, 1985) are recommended in order to increase knowledge about near-well conditions.

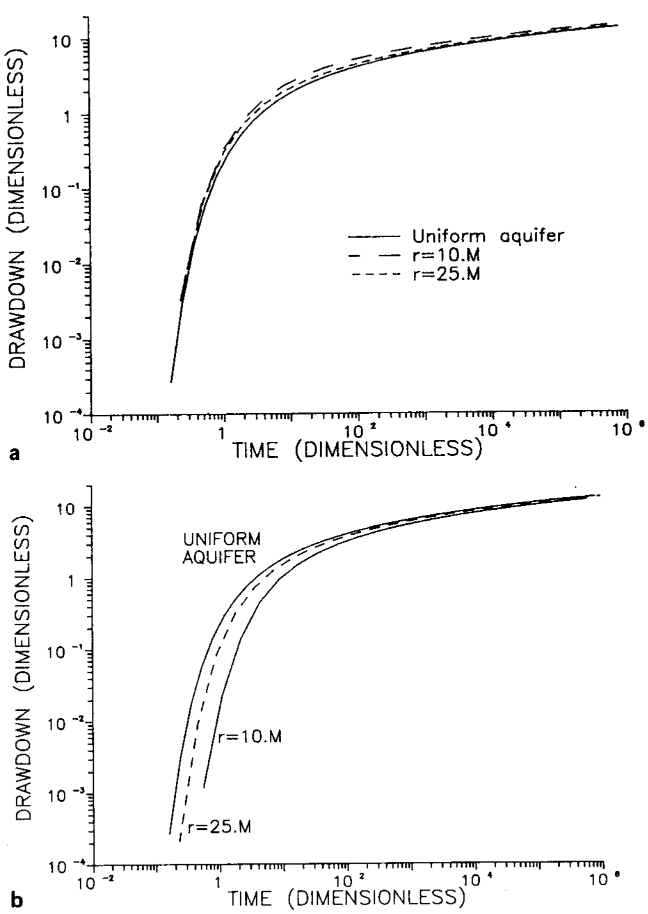


Fig. 8. Dimensionless log-log drawdown versus time plots for the analysis of the effect of near-well material on observation-well drawdown (conditions as in Scenario 3 of Table 1): a — plots for the case of an observation well sited at point D on Figure 5; and b — plots for the case of an observation well sited at point E on Figure 5.

An interesting application of this yield versus drawdown concept involves the siting of a new pumping well at one of several locations where observation wells currently exist. One can envision a scenario involving aquifer remediation where pumping wells are to be sited at a number of locations but only very limited hydraulic testing can be performed due to financial or regulatory constraints. Assume that a number of wells have been placed in an aquifer, several of which are to be employed as pumping wells. Assume also that a pumping test has been performed at one of these wells, with the other wells serving as observation points. Analysis of drawdown at an observation well provides parameters that may be of little use in estimating the drawdown at that well if it were to become a pumping well. Such an analysis, however, would provide parameters of use in estimating the yield that could be obtained at that well. If estimates of pumping-well drawdown are of interest, another source of information must be drawn upon. A slug test, which can provide data concerning the disturbed zone and adjacent near-well material (Moench and Hsieh, 1985; Sageev, 1986), is one potential source of information. A patchy aquifer model, such as that shown in Figure 4a, is a possible approach for incorporating information from a slug test in estimates of pumping-well drawdown. Since, as illustrated in Figure 4c, a patch of material centered on a pumping well only affects changes in drawdown at the well at very early times, the large-time approximation of the contribution of the patch to pumping-well drawdown can be employed in the analysis. The large-time approximation of drawdown due to a patch centered on a pumping well (s_p) can be written as:

$$s_p = Q/(2\pi T_p) \ln (R/r_w) \qquad (3)$$

where T_p = transmissivity of patch, $[L^2/T]$; R = radius of patch, [L]; and r_w = radius of pumping well, [L]. Combination of (3) with the truncated-series approximation for pumping-induced drawdown developed by Cooper and Jacob (1946) produces the following expression for pumping-well drawdown:

$$s = \frac{Q}{4\pi T} \ln \frac{2.25\,Tt}{r_w^2 S} + (s_p - \frac{Q}{2\pi T} \ln (\frac{R}{r_w}))$$

$$= \frac{Q}{4\pi T} \ln \frac{2.25\,Tt}{r_w^2 S} + \frac{Q}{2\pi T} [(\frac{T}{T_p} - 1) \ln (\frac{R}{r_w})] \qquad (4)$$

where s = drawdown at the pumping well, [L]; T = transmissivity of the aquifer, $[L^2/T]$; and S = storativity of the aquifer, dimensionless. The subtraction in the right-most term of (4) is to remove the effect of material of transmissivity T that the truncated-series equation for drawdown assumes exists between r_w and R. Note that the bracketed expression is in a form commonly used to represent the effect of drawdown due to a well skin of finite thickness (Streltsova, 1988). The use of (4) should help to increase the accuracy of estimates of drawdown at a prospective pumping well, while not influencing yield predictions. The major uncertainty concerning the method is the rather arbitrary estimate of the patch radius. In most cases, however, by employing the available slug-test data and a knowledge of the drilling method and site geology, one should be able to propose a reasonable minimum estimate for the patch radius. The resultant conservative estimate of drawdown due to near-well materials should be of use in those situations where pumping tests cannot be employed. Note that although (4) does not consider the effect of head losses within the well, it can be modified readily to account for such losses (e.g., Walton, 1970).

Conclusions

The log-log and semilog methodologies for analyzing pumping-test drawdown provide dissimilar estimates in nonuniform aquifers due to their emphasis on properties in different portions of the aquifer. The magnitude of the difference in the estimated parameters depends on the degree of aquifer nonuniformity and the distance between the observation and pumping wells. Under most conditions, this difference decreases as the distance between the observation and pumping wells increases, due to the lessening influence of local material on drawdown. An analytical solution can be used to estimate the distance above which local properties will have a negligible effect on observation-well drawdown.

There are several implications of this difference for site-characterization activities. If estimates of pumping-well yield are of interest, the semilog approach using large-time data is the appropriate analysis. Since this technique, when applied at large times, ignores the contributions of local materials, the estimated transmissivities are essentially independent of observation-well location. When estimates of future pumping-well drawdown are desired, a log-log analysis using drawdown at or very near the pumping well is the most appropriate approach. Thus, when estimates of both pumping-well drawdown and yield are of interest, an observation well at or near the pumping well should be employed. If estimates of pumping-well drawdown at a converted observation well are required, a patchy-aquifer model can be used, enabling slug-test data to be incorporated into the analysis.

The focus of this paper has been on pumping tests in rather ideal systems. Since many aquifers in nature do not behave in such an ideal fashion, the applicability of these concepts to systems more representative of actual field conditions must be considered. Although the effects of spatial variations in flow properties were discussed here, the directional dependence of these properties was not. When the anisotropy of aquifer properties is incorporated into the analysis, the concepts of this paper have considerable importance. Under anisotropic, nonuniform conditions, the effect of near-well properties can introduce considerable error into estimates of aquifer anisotropy. Therefore, wells must be placed at a considerable distance from the pumping well in order to dampen the effect of local materials. When leaky or semiconfined flow conditions are considered, the concepts discussed here must be used carefully. Under such conditions, estimates of large-scale aquifer properties may be difficult to obtain since the outward movement of the cone of depression is significantly impacted by leakage from neighboring units. In this case, estimates of aquifer properties may only be obtainable for localized areas near the pumping wells. Other techniques, such as pulse tests (Johnson *et al.*, 1966), may be more appropriate for estimating large-scale aquifer properties. Unconfined systems, on the other hand, will behave in a manner similar to that described here, although the time at which the effects of near-well material can be ignored will be much greater. Finally, it should be noted that these results pertain to flow systems for which a porous media representation is valid. Fractured systems present additional complexities that were not considered here.

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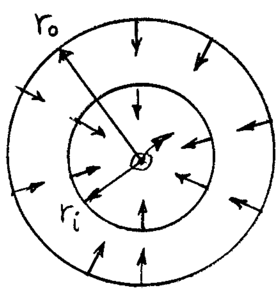
CJN
2017/09/28

NOTES ON BUTLER (1990)

I. DERIVATION OF BUTLER (1990) EQ$^{\underline{s}}$ (1) and (2)

1. Referring to Butler's figure 1, the portion of the aquifer that controls the changes in drawdown is defined as the ring between which the aquifer contributes 95% of the flow to the pumping well.



r_i : The flow across the inner radius, r_i, is 97.5 % of the pumping rate.

r_o : The flow across the outer radius, r_o, is 2.5% of the pumping rate.

The flow across any circle centered on the pumping well is:

$$Q(r) = q_r \times 2\pi r b \qquad ; \; q_r = \text{Darcy flux}, \; b = \text{aquifer thickness} \qquad \text{—(i)}$$

The proportion of the pumping that crosses a circle of radius r around the well is therefore:

$$f(r) = \frac{Q(r)}{Q}$$

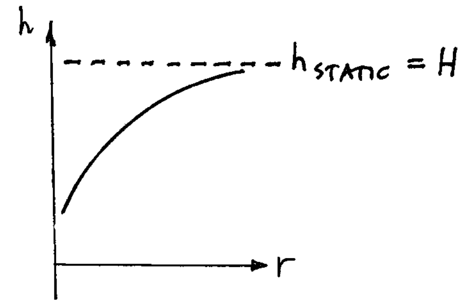
$$\therefore \quad f(r) = \frac{q_r \times 2\pi r b}{Q} \qquad \text{—(ii)}$$

The Darcy flux is derived next.

2. Derivation of the Darcy flux for the Theis solution

The Theis solution is:

$$s = \frac{Q}{4\pi T} W(u) \quad ; \quad u = \frac{r^2 S}{4Tt} \equiv \frac{r^2 S}{4Kbt}$$



From Darcy's Law:

$$q_r = -K \frac{\partial h}{\partial r}$$

Now, since $s = H - h$:

[positive Q → positive s ; flow towards well]

$$q_r = K \frac{\partial s}{\partial r}$$

$$= K \frac{\partial}{\partial r} \left[\frac{Q}{4\pi K b} W(u) \right]$$

$$= \frac{Q}{4\pi b} \frac{\partial W(u)}{\partial u} \cdot \frac{\partial u}{\partial r}$$

$$= \frac{Q}{4\pi b} \cdot \frac{2rS}{4Kbt} \frac{\partial W(u)}{\partial u}$$

$$= \frac{Q\, rS}{8\pi K b^2 t} \frac{\partial W(u)}{\partial u}$$

$$W(u) = \int_u^\infty \frac{e^{-x}}{x} dx$$

$$\therefore \frac{\partial W(u)}{\partial u} = \frac{\partial}{\partial u} \int_u^\infty \frac{e^{-x}}{x} dx$$

$$= -(1) \frac{e^{-u}}{u} = -(1) \frac{4Kbt}{r^2 S} \mathrm{EXP}\left\{ -\frac{r^2 S}{4Kbt} \right\}$$

Substituting into the expression for the Darcy flux :

$$q_r = -\frac{Q r S}{8\pi K b^2 t} \cdot \frac{4Kbt}{r^2 S} \, EXP\left\{-\frac{r^2 S}{4Kbt}\right\}$$

Simplifying :

$$\boxed{q_r = -\frac{Q}{2\pi b r} \, EXP\left\{-\frac{r^2 S}{4Tt}\right\}} \qquad T = Kb \qquad \text{—(iii)}$$

Cooper- Jacob approximation

$W(u) \cong -0.5772 - \ln u$; for $u < 0.01$

$$\therefore \frac{\partial W(u)}{\partial u} = \frac{-1}{u}$$

$$= -\frac{4Kbt}{r^2 S}$$

Substituting into the expression for the Darcy flux :

$$q_r = -\frac{Q r S}{8\pi K b^2 t} \cdot \frac{4Kbt}{r^2 S}$$

Simplifying :

$$q_r = -\frac{Q}{2\pi b r} \qquad \text{(iv)}$$

CHECK:

THE COOPER-JACOB RESULTS ARE INDEPENDENT OF TIME.
IS THERE A LARGE-TIME ASYMPTOTIC RESULT AVAILABLE WITH THE THEIS SOLUTION?

$$q_r = \frac{-Q}{2\pi b r} \exp\left\{ -\frac{r^2 S}{4Kbt} \right\}$$

$$\lim_{t\to\infty} q_r = \lim_{t\to\infty} \frac{-Q}{2\pi b r} \exp\left\{ -\frac{r^2 S}{4Kbt} \right\}$$

$$= \frac{-Q}{2\pi b r} \lim_{t\to\infty} \exp\left\{ -\frac{r^2 S}{4Kbt} \right\} \to 1$$

$$= \frac{-Q}{2\pi b r}$$

WHICH IS THE SAME AS THE COOPER-JACOB RESULT ✓

3. Returning to the expression for the fraction of the pumping that passes across a circle of radius r, substituting for q_r from (iii) into (ii) :

$$f(r) = \frac{\left[-\frac{Q}{2\pi b r} \text{EXP}\left\{ -\frac{r^2 S}{4Kbt} \right\} \right] * 2\pi r b}{Q}$$

Simplifying :

$$f(r) = \text{EXP}\left\{ -\frac{r^2 S}{4Tt} \right\}$$

The distance to where the flow is a fraction f_r is given by :

$$\boxed{r = \sqrt{4\left(-\ln\{f_r\}\right)} \left(\frac{Tt}{S}\right)^{1/2}} \qquad \text{—— (v)}$$

<u>For r_{inner}</u> :

$f_r = 0.975 \quad \therefore \quad r = \sqrt{4\left(-\ln\{0.975\}\right)} \left(\frac{T}{S} t\right)^{1/2}$

$= \sqrt{0.101} \left(\frac{Tt}{S}\right)^{1/2}$ This is EQ^n^ (1) ✓

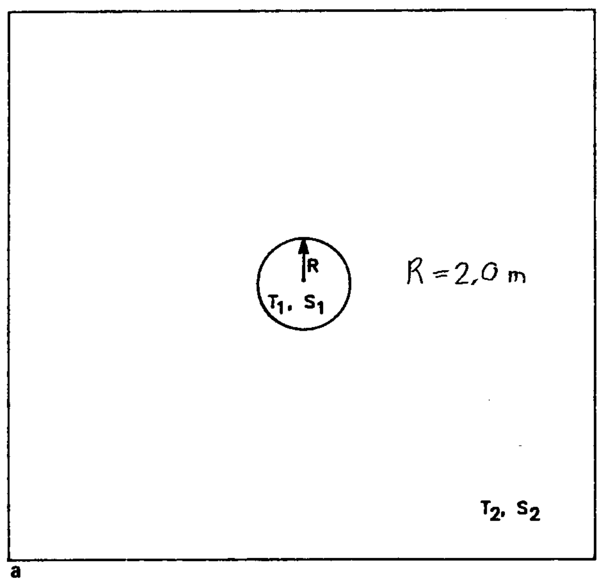
<u>For r_{outer}</u> :

$f_r = 0.025 \quad \therefore \quad r = \sqrt{4\left(-\ln\{0.025\}\right)} \left(\frac{T}{S} t\right)^{1/2}$

$= \sqrt{14.756} \left(\frac{Tt}{S}\right)^{1/2}$ This is EQ^n^ (2) ✓

II. BUTLER (1990) EXAMPLES

1. EX #1 (Figure 4)



$T_1 = 0.1\,T_2$

$S_1 = S_2$

$T_1 = 0.1\, T_2$

$R = 2.0\, m$

$r = 0.05\, m$

↳ with disk of Material 1

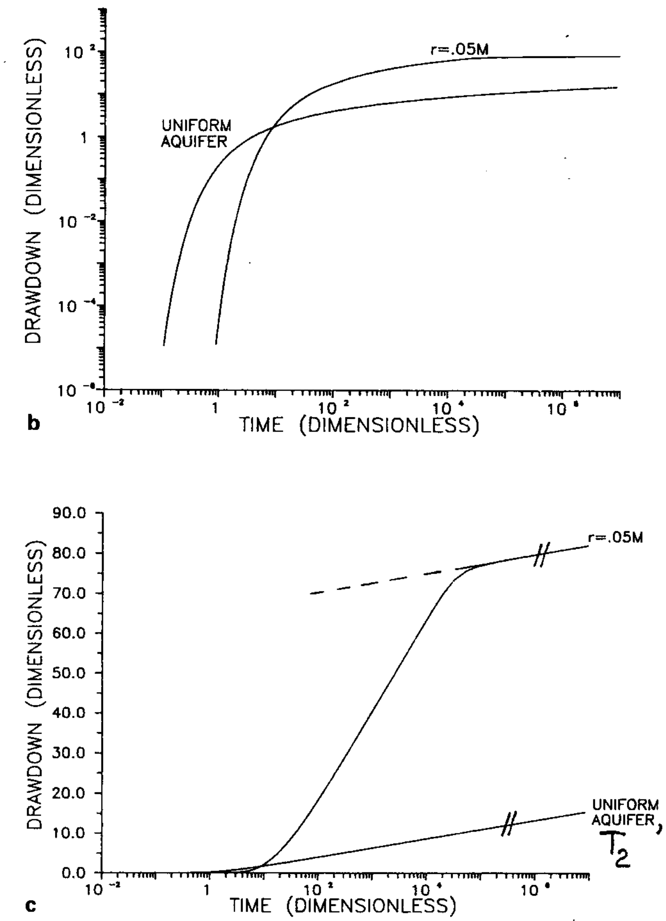


Fig. 4. a — Configuration consisting of a disk of material 1 (R = 2 m) embedded in material 2 (T_1/T_2 = .1, S_1/S_2 = 1.0); b — dimensionless log-log drawdown ($4\pi T_2 s/Q$) versus time ($4T_2 t/r^2 S_2$) plots for the embedded-disk and uniform-aquifer cases (r = radial distance from the center of the disk to the observation point, in this case can be considered the radius of a well centered at the origin); and c — dimensionless semilog drawdown versus time plots for the embedded-disk and uniform-aquifer cases.

The drawdown at the observation well exceeds the drawdown for uniform T_2. However, after a certain time has elapsed the rate at which the drawdown increases is the same as for a uniform medium.

$$t_D = \frac{4T_2 t}{r^2 S_2} = \frac{1}{u} \ (\text{with } T_2, S_2)$$

$$s_D = \frac{4\pi T_2}{Q} s$$

2. EX *3 (Fig 5, Fig 6b, Fig 7)

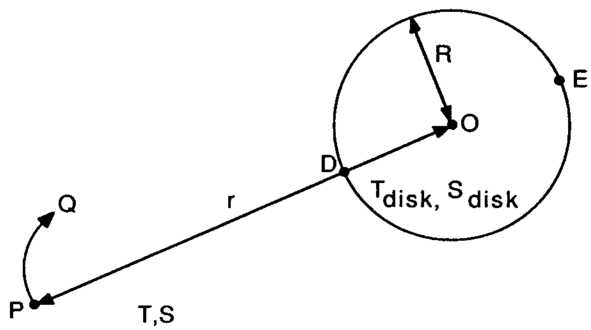


Fig. 5. Configuration employed in the analysis of the effect of near-well material on observation-well drawdown, notation explained in the text.

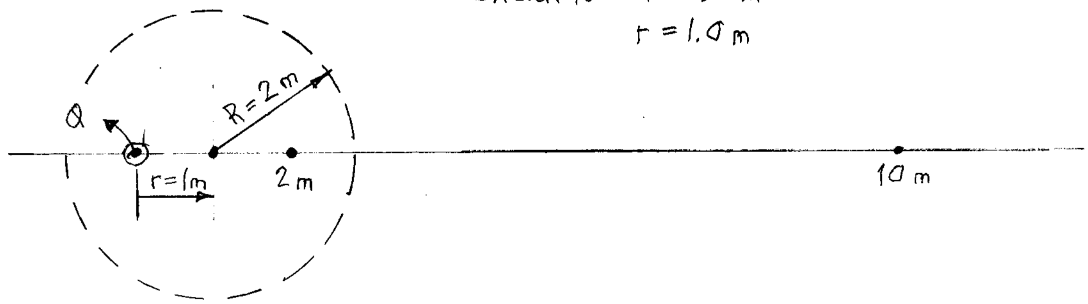
$T_{disk} = 0.1T$

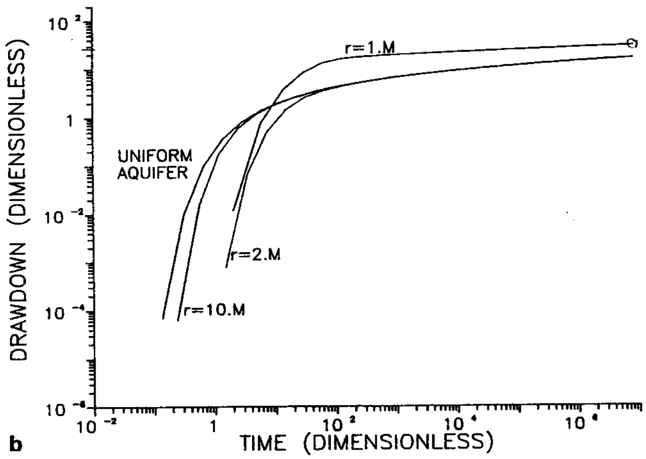
$S_{disk} = S$

cf. Butler and Liu (1989)

The observation well is located a distance r from the pumping well.

Sketch for R = 2.0 m
r = 1.0 m





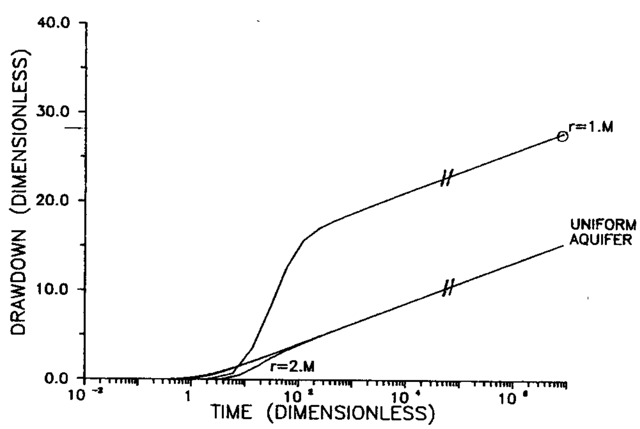


Fig. 7. Dimensionless semilog drawdown versus time plots corresponding to Figure 6b.

Curve for r = 10 m plots on top of curve for uniform aquifer